

---

**Problem 4)** Splitting each term of the series into simple fractions, then eliminating equal and opposite terms, we will have

$$\begin{aligned} S &= \frac{1}{1 \times 5} - \frac{1}{2 \times 6} + \frac{1}{3 \times 7} - \frac{1}{4 \times 8} + \dots - \frac{(-1)^n}{n \times (n+4)} + \dots \\ &= \frac{1}{4} \left[ \left( \frac{1}{1} - \frac{1}{5} \right) - \left( \frac{1}{2} - \frac{1}{6} \right) + \left( \frac{1}{3} - \frac{1}{7} \right) - \left( \frac{1}{4} - \frac{1}{8} \right) + \dots - (-1)^n \left( \frac{1}{n} - \frac{1}{n+4} \right) + \dots \right] \\ &= \frac{1}{4} \left[ \left( \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} \dots \right) - \left( \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \dots \right) \right] \\ &= \frac{1}{4} \left( \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \right) = \frac{7}{48}. \end{aligned}$$

---