Problem 2) The formula is clearly valid for $N = 1$. Suppose it holds for the integer $N \geq 1$. Then, for the sum extending from 1 to $N + 1$, we will have

$$\sum_{n=1}^{N+1} n^3 = \sum_{n=1}^{N} n^3 + (N+1)^3 = [N(N+1)/2]^2 + (N+1)^3$$

$$= [(N+1)/2]^2 \times [N^2 + 4(N+1)] = [(N+1)/2]^2 \times (N+2)^2$$

$$= [(N+1)(N+2)/2]^2.$$

Clearly, the above formula is what one gets from $[N(N+1)/2]^2$ upon substituting $N + 1$ for N, which completes our proof by induction.