Problem 1) Multiply both sides of the equation by $1 - z$, and note that $lim_{n\to\infty} z^{2^n} = 0$.