Solution to Problem 1) a) The slope of the function $y = ax^2$ at $x = x_0$, namely, $y' = 2ax_0$, is the tangent of the angle φ that the unit-vector S makes with the x-axis. Therefore,

$$\tan\varphi = 2ax_0.$$

$$\cos\varphi = 1/\sqrt{1+\tan^2\varphi} = 1/\sqrt{1+4a^2x_0^2}.$$

$$\sin\varphi = \tan\varphi\cos\varphi = 2ax_0/\sqrt{1+4a^2x_0^2}.$$

$$\boldsymbol{S} = \cos\varphi\,\hat{\boldsymbol{x}} + \sin\varphi\,\hat{\boldsymbol{y}} = (\hat{\boldsymbol{x}} + 2ax_0\hat{\boldsymbol{y}})/\sqrt{1+4a^2x_0^2}.$$

$$\boldsymbol{R}_2 = [-x_0\hat{\boldsymbol{x}} + (y_1 - y_0)\hat{\boldsymbol{y}}]/\sqrt{x_0^2 + (y_1 - y_0)^2}.$$

b) The dot-product of $\boldsymbol{S}$ and $\boldsymbol{R}_1$ gives the cosine of the angle between these two unit-vectors. Similarly, the cosine of the angle between $\boldsymbol{S}$ and $\boldsymbol{R}_2$ is given by $\boldsymbol{S}\cdot\boldsymbol{R}_2$. Given that $\boldsymbol{R}_1 = -\hat{\boldsymbol{y}}$, and that $\boldsymbol{R}_2$ and $\boldsymbol{S}$ are found in part (a), we now set $\boldsymbol{S}\cdot\boldsymbol{R}_1 = \boldsymbol{S}\cdot\boldsymbol{R}_2$. Consequently, the various system parameters are related as follows:

$$-2ax_0 = [-x_0 + 2ax_0(y_1 - y_0)]/\sqrt{x_0^2 + (y_1 - y_0)^2}.$$

c) The above equation may now be solved to determine y_1 in terms of the remaining parameters. Recalling that $y_0 = ax_0^2$, we will have

$$2a = [1 - 2a(y_1 - ax_0^2)]/\sqrt{x_0^2 + (y_1 - ax_0^2)^2}$$

$$\rightarrow \quad 4a^2[x_0^2 + (y_1 - ax_0^2)^2] = 1 - 4a(y_1 - ax_0^2) + 4a^2(y_1 - ax_0^2)^2$$

$$\rightarrow \quad 4a^2x_0^2 = 1 - 4ay_1 + 4a^2x_0^2 \quad \rightarrow \quad y_1 = 1/(4a).$$