Opti 503 **Midterm Exam** (4/2/2012) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

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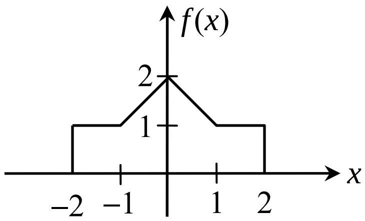
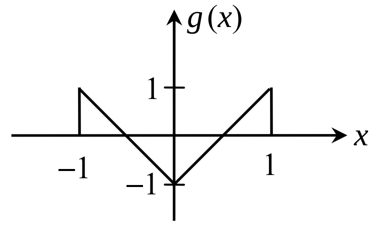
5 pts **Problem 1**) Find the Fourier transforms of the functions $f(x)$ and $g(x)$ shown below.





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1 pt **Problem 2**) a) Show that the Fourier transform of $\cos(2\pi f_o x)$ is $\tfrac{1}{2}[\delta(s-f_o)+\delta(s+f_o)]$.

1 pt b) Show that the Fourier transform of $\sin(2\pi f_o x)$ is $[\delta(s-f_o)-\delta(s+f_o)]/(2\mathrm{i})$.

2 pts c) Using the result of part (a), find the Fourier transform of $\cos^2(\pi f_o x)$.

2 pts d) Use the differentiation theorem to find the Fourier transform of the derivative of $\cos^2(\pi f_o x)$. Show that your final result is consistent with the result obtained in part (b).

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5 pts **Problem 3**) Show that $\int_{-\infty}^{\infty}\frac{\cos x}{\pi^2-4x^2}\mathrm{d}x=\frac{1}{2}$. (Note that the integrand does *not* diverge at $x=\pm\pi/2$, as the numerator, $\cos x$, also goes to zero at these points.)

Hint: Split the integral into two separate integrals using the identity $\cos x=\tfrac{1}{2}[\exp(\mathrm{i}x)+\exp(-\mathrm{i}x)]$.

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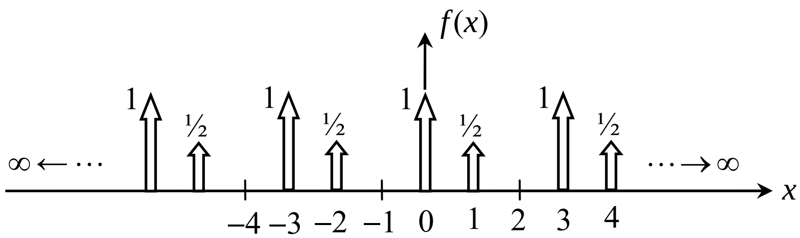
5 pts **Problem 4**) Use two different methods to find the Fourier transform of the periodic function $f(x)$ shown below. Confirm that both methods yield the same answer.



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4 pts **Problem 5**) Determine an approximate value for the following integral, assuming that the real-valued parameter η is large and positive.

$$I=\int_{-1}^{+1}\exp(-x^2+\mathrm{i}\eta\sqrt{1-x^2})\,\mathrm{d}x.$$
