Opti 503 **Midterm Exam** (2/22/2012) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

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5 pts **Problem 1**) Show that $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{(n+1)^2}=\frac{\pi^2}{12}-\ln 2.$

Hint: Use the fact that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=\frac{\pi^2}{12}$ and also $\ln(2)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}.$

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5 pts **Problem 2**) Show that $\sum_{n=1}^{\infty}nx^n=\frac{x}{(1-x)^2}$, where $|x|<1$.

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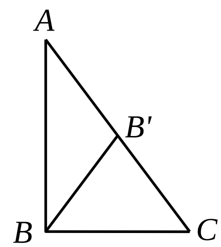
5 pts **Problem 3**) In a right triangle ABC, the median BB' splits the hypotenuse AC into equal halves, that is, $AB'=CB'$. Show that the length of this median is half the length of the hypotenuse, that is $BB'=\frac{1}{2}AC$.



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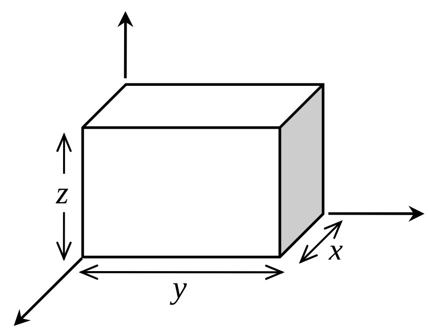
5 pts **Problem 4**) A house in the shape of a cuboid is to be built on a flat piece of land, as shown. The volume of the house $V=xyz$ is desired to be some constant value, V_o. However, the exposed surface area (that is, the exterior walls plus the roof) should be minimized. Use the method of Lagrange multipliers to determine the optimal dimensions of the house in terms of its desired volume V_o.



Problem 5) Use the Cauchy-Riemann conditions to determine the domain of analyticity of each of the following functions:

2 pts a) $f_1(z)=\frac{1}{z-z_o}$;

3 pts b) $f_2(z)=\exp(z^2)$.
