Opti 503A **Final Exam** (5/12/2014) **Time: 2 hours**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

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25 pts **Problem 1**) Use residue calculus to show that

$$\int_0^\infty \frac{\mathrm{d}x}{(x^2+a^2)^2} = \frac{\pi}{4a^3}; \qquad a > 0.$$

Hint: Both poles are second-order poles.

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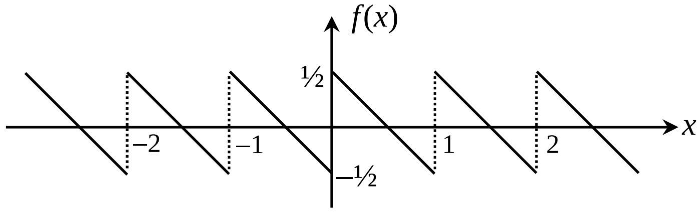
15 pts **Problem 2**) a) Find the Fourier series representation of the periodic function $f(x)$, having period $P = 1$, and defined over the interval $0 \le x < 1$ as $f(x) = ½ - x$.



10 pts b) Differentiating $f(x)$ and its Fourier series representation (term by term) with respect to x, show that the periodic function $f'(x)$ is indeed equal to the derivative of the Fourier series representation of $f(x)$.

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20 pts **Problem 3**) Use the method of Frobenius to find the solutions of Airy's differential equation, namely,

$$f''(x) - xf(x) = 0.$$

Hint: There are four solutions to the indicial equations, but Airy's differential equation has only two independent solutions.

Problem 4) A homogeneous medium is specified by its specific heat C, thermal conductivity κ, and thermal diffusivity D. The temperature distribution in this medium in a cylindrical coordinate system is denoted by the function $T(r, \varphi, z, t)$.

15 pts a) Derive the general form of the differential equation that governs the temperature distribution in space and its evolution in time. Show all the steps, starting with a physical explanation of the procedure and ending with the complete diffusion equation in cylindrical coordinates.

10 pts b) Use the method of separation of variables to find the general solution for $T(r, \varphi, z, t)$.

5 pts c) Explain how the various unknown parameters can be obtained with the help of the boundary and initial conditions.
