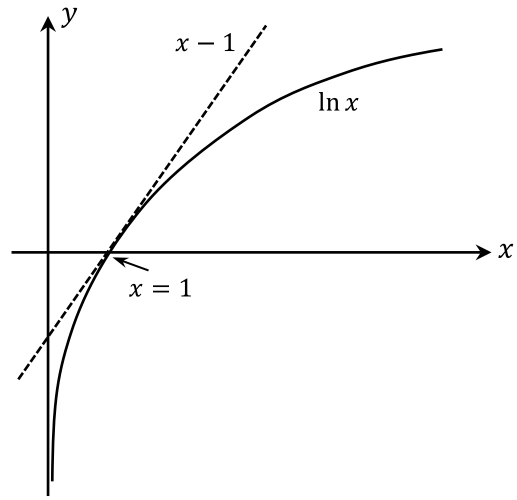
Solution to Problem 11) a) The plot of the function $\ln x$, shown below, is tangent to the straight line $y = x - 1$, which makes contact with $\ln x$ at $x = 1$. Since $\ln x$ is convex cap throughout the entire region $x > 0$, it must stay below its tangent line. Consequently, $\ln x \le x - 1$, with equality satisfied only at $x = 1$.



b) $I(x, y) = H(x) - H(x|y)$

$$= -\sum_n p_x(x_n) \ln p_x(x_n) + \sum_m p_y(y_m) \sum_n \left[p_{x|y}(x_n|y_m) \ln p_{x|y}(x_n|y_m)\right]$$

$$= -\sum_m \sum_n p_{xy}(x_n, y_m) \ln p_x(x_n) + \sum_m \sum_n \left[p_y(y_m) p_{x|y}(x_n|y_m) \ln p_{x|y}(x_n|y_m)\right]$$

$$= -\sum_m \sum_n p_{xy}(x_n, y_m) \ln[p_x(x_n) p_y(y_m)/p_{xy}(x_n, y_m)]$$

$$\ge \sum_m \sum_n p_{xy}(x_n, y_m)\{1 - [p_x(x_n) p_y(y_m)/p_{xy}(x_n, y_m)]\}$$

$$= \sum_m \sum_n p_{xy}(x_n, y_m) - \sum_m \sum_n p_x(x_n) p_y(y_m)$$

$$= 1 - [\sum_n p_x(x_n)] \times [\sum_m p_y(y_m)] = 1 - 1 \times 1 = 0.$$

Therefore, $I(x, y) \ge 0$. Equality holds only if x and y happen to be independent random variables, that is, when $p_{xy}(x_n, y_m) = p_x(x_n) p_y(y_m)$ for all values of n and m.