Solution to Problem 9) With reference to Problem 8, the characteristic functions of our two (independent) random variables x_1 and x_2 are given by

$$\psi_1(s) = \exp[\alpha_1 \cos(2\pi s) - \alpha_1 - i\alpha_1 \sin(2\pi s)].$$

$$\psi_2(s) = \exp[\alpha_2 \cos(2\pi s) - \alpha_2 - i\alpha_2 \sin(2\pi s)].$$

The characteristic function of the sum $x_1 + x_2$ of these independent random variables is thus given by $\psi_1(s)\psi_2(s) = \exp[(\alpha_1 + \alpha_2)\cos(2\pi s) - (\alpha_1 + \alpha_2) - i(\alpha_1 + \alpha_2)\sin(2\pi s)]$. Clearly, the sum $x_1 + x_2$ has a Poisson distribution with the parameter $\alpha_1 + \alpha_2$.