Solution to Problem 7) With reference to Problem 6, the characteristic functions of our binomial distributions are given by

$$\psi_1(s) = [p\cos(2\pi s) + 1 - p - \mathrm{i}p\sin(2\pi s)]^{N_1}.$$

$$\psi_2(s) = [p\cos(2\pi s) + 1 - p - \mathrm{i}p\sin(2\pi s)]^{N_2}.$$

Upon multiplying the above characteristic functions, we find the characteristic function of the sum $x_1 + x_2$ of our two random variables. Considering that both x_1 and x_2 have the same parameter p, the product of their characteristic functions will be that of a binomial distribution with parameters p and $N_1 + N_2$.

The above result, of course, should have been anticipated from the basic properties of the binomial distribution. Each binomial distribution is the result of N repetitions of a binary experiment whose outcomes, $\zeta_1 = 1$ and $\zeta_2 = 0$, have probabilities $p_1 = p$ and $p_2 = (1 - p)$, respectively. If the binary experiment is repeated N_1 (independent) times and its outcomes are added together, we obtain our first random variable x_1. Similarly, if the binary experiment is repeated N_2 times (again independently) and its outcomes are added together, we obtain our second random variable x_2. It is clear that adding x_1 and x_2 is tantamount to repeating the original binary experiment (with outcomes ζ_1 and ζ_2) a total of $N_1 + N_2$ times.
