**Solution to Problem 3**) Let $z = x + y$ be the sum of two independent random variables, which are denoted by $x$ and $y$. The average, the second moment, and the variance of $z$ are readily found as follows:

$$\langle z \rangle = \langle x + y \rangle = \langle x \rangle + \langle y \rangle.$$

$$\langle z^2 \rangle = \langle (x + y)^2 \rangle = \langle x^2 + y^2 + 2xy \rangle = \langle x^2 \rangle + \langle y^2 \rangle + 2\langle x \rangle \langle y \rangle.$$

$$\sigma_z^2 = \langle z^2 \rangle - \langle z \rangle^2 = [\langle x^2 \rangle + \langle y^2 \rangle + 2\langle x \rangle \langle y \rangle] - [\langle x \rangle^2 + \langle y \rangle^2 + 2\langle x \rangle \langle y \rangle] = \sigma_x^2 + \sigma_y^2.$$

The same results can be obtained by recognizing that the characteristic function $\psi_z(s)$ of $z$ is the product of the characteristic functions $\psi_x(s)$ and $\psi_y(s)$ of the random variables $x$ and $y$. We then write

$$\psi_z'(s)|_{s=0} = [\psi_x'(s)\psi_y(s) + \psi_x(s)\psi_y'(s)]\big|_{s=0} = \psi_x'(0) + \psi_y'(0).$$

$$\psi_z''(s)|_{s=0} = [\psi_x''(s)\psi_y(s) + \psi_x(s)\psi_y''(s) + 2\psi_x'(s)\psi_y'(s)]\big|_{s=0}$$

$$= \psi_x''(0) + \psi_y''(0) + 2\psi_x'(0)\psi_y'(0).$$

From Eq.(29), Sec.7, we learn that $\psi_x'(0) = -\mathrm{i}2\pi\langle x \rangle$ and $\psi_y'(0) = -\mathrm{i}2\pi\langle y \rangle$. Consequently, $\psi_z'(0) = -\mathrm{i}2\pi(\langle x \rangle + \langle y \rangle)$, which yields $\langle z \rangle = \langle x \rangle + \langle y \rangle$. Similarly, $\psi_x''(0) = -4\pi^2\langle x^2 \rangle$ and $\psi_y''(0) = -4\pi^2\langle y^2 \rangle$, which result in

$$\psi_z''(0) = -4\pi^2(\langle x^2 \rangle + \langle y^2 \rangle + 2\langle x \rangle\langle y \rangle) \quad \rightarrow \quad \langle z^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + 2\langle x \rangle\langle y \rangle.$$

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