

Solution to Problem 2) Since $h(x)$ is, by definition, the convolution of the functions $f(x)$ and $g(x)$, we have

$$h(x) = \int_{-\infty}^{\infty} f(x')g(x - x')dx'$$

Integrating both sides of the above equation yields

$$\begin{aligned}\int_{-\infty}^{\infty} h(x)dx &= \iint_{-\infty}^{\infty} f(x')g(x - x')dx dx' \\ &= \int_{-\infty}^{\infty} f(x')\left[\int_{-\infty}^{\infty} g(x - x')dx\right]dx' \leftarrow \text{change of variable } x'' = x - x' \\ &= \int_{-\infty}^{\infty} f(x')dx' \times \int_{-\infty}^{\infty} g(x'')dx''\end{aligned}$$

The area under the function $h(x)$ is thus seen to be the product of the areas under $f(x)$ and $g(x)$. The same result may be obtained by examining the Fourier transforms $F(s)$, $G(s)$, and $H(s)$ of the functions under consideration. Since $H(s) = F(s)G(s)$, it is straightforward to observe that the area under $h(x)$, which is given by $H(0)$, equals the product $F(0)G(0)$ of the areas under the functions $f(x)$ and $g(x)$.
