

12.24) Considering that $\rho = \sqrt{x^2 + y^2}$, $\hat{\rho} = \cos \phi \hat{x} + \sin \phi \hat{y}$, $\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}$, $\sin \phi = y/\sqrt{x^2 + y^2}$, and $\cos \phi = x/\sqrt{x^2 + y^2}$, we may write

$$\mathbf{A}(\mathbf{r}) = (A_\rho \cos \phi - A_\phi \sin \phi)\hat{x} + (A_\rho \sin \phi + A_\phi \cos \phi)\hat{y} + A_z\hat{z},$$

$$\mathbf{B}(\mathbf{r}) = (B_\rho \cos \phi - B_\phi \sin \phi)\hat{x} + (B_\rho \sin \phi + B_\phi \cos \phi)\hat{y} + B_z\hat{z},$$

$$\frac{\partial \rho}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{\rho} = \cos \phi,$$

$$\frac{\partial \rho}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{\rho} = \sin \phi,$$

$$\frac{\partial \sin \phi}{\partial x} = -\frac{xy}{(x^2 + y^2)^{3/2}} = -\frac{xy}{\rho^3} = -\frac{\sin \phi \cos \phi}{\rho} \rightarrow \left(\frac{\partial \phi}{\partial x}\right) \cos \phi = -\frac{xy}{\rho^3} \rightarrow \frac{\partial \phi}{\partial x} = -\frac{y}{\rho^2} = -\frac{\sin \phi}{\rho},$$

$$\frac{\partial \sin \phi}{\partial y} = \frac{x^2}{(x^2 + y^2)^{3/2}} = \frac{x^2}{\rho^3} = \frac{\cos^2 \phi}{\rho} \rightarrow \left(\frac{\partial \phi}{\partial y}\right) \cos \phi = \frac{x^2}{\rho^3} \rightarrow \frac{\partial \phi}{\partial y} = \frac{x}{\rho^2} = \frac{\cos \phi}{\rho},$$

$$\frac{\partial \cos \phi}{\partial x} = \frac{y^2}{(x^2 + y^2)^{3/2}} = \frac{y^2}{\rho^3} = \frac{\sin^2 \phi}{\rho},$$

$$\frac{\partial \cos \phi}{\partial y} = -\frac{xy}{(x^2 + y^2)^{3/2}} = -\frac{xy}{\rho^3} = -\frac{\sin \phi \cos \phi}{\rho}.$$

We thus have

$$\begin{aligned} A_x \frac{\partial B_x}{\partial x} &= (A_\rho \cos \phi - A_\phi \sin \phi) \left(\frac{\partial B_\rho}{\partial x} \cos \phi + \frac{\sin^2 \phi}{\rho} B_\rho - \frac{\partial B_\phi}{\partial x} \sin \phi + \frac{\sin \phi \cos \phi}{\rho} B_\phi \right) \\ &= (A_\rho \cos \phi - A_\phi \sin \phi) \\ &\quad \times \left[\left(\frac{\partial B_\rho}{\partial \rho} \frac{\partial \rho}{\partial x} + \frac{\partial B_\rho}{\partial \phi} \frac{\partial \phi}{\partial x} \right) \cos \phi + \frac{\sin^2 \phi}{\rho} B_\rho - \left(\frac{\partial B_\phi}{\partial \rho} \frac{\partial \rho}{\partial x} + \frac{\partial B_\phi}{\partial \phi} \frac{\partial \phi}{\partial x} \right) \sin \phi + \frac{\sin \phi \cos \phi}{\rho} B_\phi \right] \\ &= (A_\rho \cos \phi - A_\phi \sin \phi) \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\rho}{\rho \partial \phi} \sin \phi \right) \cos \phi + \frac{\sin^2 \phi}{\rho} B_\rho \right. \\ &\quad \left. - \left(\frac{\partial B_\phi}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi + \frac{\sin \phi \cos \phi}{\rho} B_\phi \right] \\ &= A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \cos^2 \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi \cos \phi \right] \\ &\quad - A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi \cos \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin^2 \phi \right] \\ &\quad + \left(\frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\rho + \frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\phi - \frac{\sin^3 \phi}{\rho} A_\phi B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\phi B_\phi \right). \end{aligned}$$

$$\begin{aligned} A_x \frac{\partial B_y}{\partial x} &= (A_\rho \cos \phi - A_\phi \sin \phi) \left(\frac{\partial B_\rho}{\partial x} \sin \phi - \frac{\sin \phi \cos \phi}{\rho} B_\rho + \frac{\partial B_\phi}{\partial x} \cos \phi + \frac{\sin^2 \phi}{\rho} B_\phi \right) \\ &= (A_\rho \cos \phi - A_\phi \sin \phi) \\ &\quad \times \left[\left(\frac{\partial B_\rho}{\partial \rho} \frac{\partial \rho}{\partial x} + \frac{\partial B_\rho}{\partial \phi} \frac{\partial \phi}{\partial x} \right) \sin \phi - \frac{\sin \phi \cos \phi}{\rho} B_\rho + \left(\frac{\partial B_\phi}{\partial \rho} \frac{\partial \rho}{\partial x} + \frac{\partial B_\phi}{\partial \phi} \frac{\partial \phi}{\partial x} \right) \cos \phi + \frac{\sin^2 \phi}{\rho} B_\phi \right] \end{aligned}$$

$$\begin{aligned}
&= (A_\rho \cos \phi - A_\phi \sin \phi) \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\rho}{\rho \partial \phi} \sin \phi \right) \sin \phi - \frac{\sin \phi \cos \phi}{\rho} B_\rho \right. \\
&\quad \left. + \left(\frac{\partial B_\phi}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \cos \phi + \frac{\sin^2 \phi}{\rho} B_\phi \right] \\
&= A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \cos^2 \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin \phi \cos \phi \right] \\
&\quad - A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin \phi \cos \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin^2 \phi \right] \\
&\quad - \left(\frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\phi - \frac{\sin^2 \phi \cos \phi}{\rho} A_\phi B_\rho + \frac{\sin^3 \phi}{\rho} A_\phi B_\phi \right).
\end{aligned}$$

$$\begin{aligned}
A_x \frac{\partial B_z}{\partial x} &= (A_\rho \cos \phi - A_\phi \sin \phi) \left(\frac{\partial B_z}{\partial \rho} \cos \phi - \frac{\partial B_z}{\rho \partial \phi} \sin \phi \right) \\
&= A_\rho \left(\frac{\partial B_z}{\partial \rho} \cos \phi - \frac{\partial B_z}{\rho \partial \phi} \sin \phi \right) \cos \phi - A_\phi \left(\frac{\partial B_z}{\partial \rho} \cos \phi - \frac{\partial B_z}{\rho \partial \phi} \sin \phi \right) \sin \phi.
\end{aligned}$$

$$\begin{aligned}
A_y \frac{\partial B_x}{\partial y} &= (A_\rho \sin \phi + A_\phi \cos \phi) \left(\frac{\partial B_\rho}{\partial y} \cos \phi - \frac{\sin \phi \cos \phi}{\rho} B_\rho - \frac{\partial B_\phi}{\partial y} \sin \phi - \frac{\cos^2 \phi}{\rho} B_\phi \right) \\
&= (A_\rho \sin \phi + A_\phi \cos \phi) \\
&\quad \times \left[\left(\frac{\partial B_\rho}{\partial \rho} \frac{\partial \rho}{\partial y} + \frac{\partial B_\rho}{\partial \phi} \frac{\partial \phi}{\partial y} \right) \cos \phi - \frac{\sin \phi \cos \phi}{\rho} B_\rho - \left(\frac{\partial B_\phi}{\partial \rho} \frac{\partial \rho}{\partial y} + \frac{\partial B_\phi}{\partial \phi} \frac{\partial \phi}{\partial y} \right) \sin \phi - \frac{\cos^2 \phi}{\rho} B_\phi \right] \\
&= (A_\rho \sin \phi + A_\phi \cos \phi) \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\rho}{\rho \partial \phi} \cos \phi \right) \cos \phi - \frac{\sin \phi \cos \phi}{\rho} B_\rho \right. \\
&\quad \left. - \left(\frac{\partial B_\phi}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin \phi - \frac{\cos^2 \phi}{\rho} B_\phi \right] \\
&= A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin^2 \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi \cos \phi \right] \\
&\quad + A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin \phi \cos \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \cos^2 \phi \right] \\
&\quad - \left(\frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\rho + \frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\phi + \frac{\sin \phi \cos^2 \phi}{\rho} A_\phi B_\rho + \frac{\cos^3 \phi}{\rho} A_\phi B_\phi \right).
\end{aligned}$$

$$\begin{aligned}
A_y \frac{\partial B_y}{\partial y} &= (A_\rho \sin \phi + A_\phi \cos \phi) \left(\frac{\partial B_\rho}{\partial y} \sin \phi + \frac{\cos^2 \phi}{\rho} B_\rho + \frac{\partial B_\phi}{\partial y} \cos \phi - \frac{\sin \phi \cos \phi}{\rho} B_\phi \right) \\
&= (A_\rho \sin \phi + A_\phi \cos \phi) \\
&\quad \times \left[\left(\frac{\partial B_\rho}{\partial \rho} \frac{\partial \rho}{\partial y} + \frac{\partial B_\rho}{\partial \phi} \frac{\partial \phi}{\partial y} \right) \sin \phi + \frac{\cos^2 \phi}{\rho} B_\rho + \left(\frac{\partial B_\phi}{\partial \rho} \frac{\partial \rho}{\partial y} + \frac{\partial B_\phi}{\partial \phi} \frac{\partial \phi}{\partial y} \right) \cos \phi - \frac{\sin \phi \cos \phi}{\rho} B_\phi \right] \\
&= (A_\rho \sin \phi + A_\phi \cos \phi) \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\rho}{\rho \partial \phi} \cos \phi \right) \sin \phi + \frac{\cos^2 \phi}{\rho} B_\rho \right. \\
&\quad \left. + \left(\frac{\partial B_\phi}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \cos \phi - \frac{\sin \phi \cos \phi}{\rho} B_\phi \right] \\
&= A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin^2 \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin \phi \cos \phi \right]
\end{aligned}$$

$$\begin{aligned}
& + A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin \phi \cos \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \cos^2 \phi \right] \\
& + \left(\frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\phi + \frac{\cos^3 \phi}{\rho} A_\phi B_\rho - \frac{\sin \phi \cos^2 \phi}{\rho} A_\phi B_\phi \right).
\end{aligned}$$

$$\begin{aligned}
A_y \frac{\partial B_z}{\partial y} &= (A_\rho \sin \phi + A_\phi \cos \phi) \left(\frac{\partial B_z}{\partial \rho} \sin \phi + \frac{\partial B_z}{\rho \partial \phi} \cos \phi \right) \\
&= A_\rho \left(\frac{\partial B_z}{\partial \rho} \sin \phi + \frac{\partial B_z}{\rho \partial \phi} \cos \phi \right) \sin \phi + A_\phi \left(\frac{\partial B_z}{\partial \rho} \sin \phi + \frac{\partial B_z}{\rho \partial \phi} \cos \phi \right) \cos \phi.
\end{aligned}$$

$$A_z \frac{\partial B_x}{\partial z} = A_z \left(\frac{\partial B_\rho}{\partial z} \cos \phi - \frac{\partial B_\phi}{\partial z} \sin \phi \right),$$

$$A_z \frac{\partial B_y}{\partial z} = A_z \left(\frac{\partial B_\rho}{\partial z} \sin \phi + \frac{\partial B_\phi}{\partial z} \cos \phi \right),$$

$$A_z \frac{\partial B_z}{\partial z} = A_z \frac{\partial B_z}{\partial z}.$$

Combining the above expressions, we find

$$\begin{aligned}
(\mathbf{A} \cdot \nabla) \mathbf{B} &= A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \cos^2 \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi \cos \phi \right] \hat{\mathbf{x}} \\
&- A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin \phi \cos \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin^2 \phi \right] \hat{\mathbf{x}} \\
&+ \left(\frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\rho + \frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\phi - \frac{\sin^3 \phi}{\rho} A_\phi B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\phi B_\phi \right) \hat{\mathbf{x}} \\
&+ A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \cos^2 \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin \phi \cos \phi \right] \hat{\mathbf{y}} \\
&- A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin \phi \cos \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin^2 \phi \right] \hat{\mathbf{y}} \\
&- \left(\frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\phi - \frac{\sin^2 \phi \cos \phi}{\rho} A_\phi B_\rho + \frac{\sin^3 \phi}{\rho} A_\phi B_\phi \right) \hat{\mathbf{y}} \\
&+ \left[A_\rho \left(\frac{\partial B_z}{\partial \rho} \cos \phi - \frac{\partial B_z}{\rho \partial \phi} \sin \phi \right) \cos \phi - A_\phi \left(\frac{\partial B_z}{\partial \rho} \cos \phi - \frac{\partial B_z}{\rho \partial \phi} \sin \phi \right) \sin \phi \right] \hat{\mathbf{z}} \\
&+ A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin^2 \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi \cos \phi \right] \hat{\mathbf{x}} \\
&+ A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin \phi \cos \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \cos^2 \phi \right] \hat{\mathbf{x}} \\
&- \left(\frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\rho + \frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\phi + \frac{\sin \phi \cos^2 \phi}{\rho} A_\phi B_\rho + \frac{\cos^3 \phi}{\rho} A_\phi B_\phi \right) \hat{\mathbf{x}} \\
&+ A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin^2 \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin \phi \cos \phi \right] \hat{\mathbf{y}} \\
&+ A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin \phi \cos \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \cos^2 \phi \right] \hat{\mathbf{y}} \\
&+ \left(\frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\phi + \frac{\cos^3 \phi}{\rho} A_\phi B_\rho - \frac{\sin \phi \cos^2 \phi}{\rho} A_\phi B_\phi \right) \hat{\mathbf{y}}
\end{aligned}$$

$$\begin{aligned}
& + \left[A_\rho \left(\frac{\partial B_z}{\partial \rho} \sin \phi + \frac{\partial B_z}{\rho \partial \phi} \cos \phi \right) \sin \phi + A_\phi \left(\frac{\partial B_z}{\partial \rho} \sin \phi + \frac{\partial B_z}{\rho \partial \phi} \cos \phi \right) \cos \phi \right] \hat{\mathbf{z}} \\
& + A_z \left(\frac{\partial B_\rho}{\partial z} \cos \phi - \frac{\partial B_\phi}{\partial z} \sin \phi \right) \hat{\mathbf{x}} + A_z \left(\frac{\partial B_\rho}{\partial z} \sin \phi + \frac{\partial B_\phi}{\partial z} \cos \phi \right) \hat{\mathbf{y}} + A_z \frac{\partial B_z}{\partial z} \hat{\mathbf{z}}.
\end{aligned}$$

Further simplifications yield

$$\begin{aligned}
(\mathbf{A} \cdot \nabla) \mathbf{B} = & \left\{ A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \cos^2 \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi \cos \phi \right] \right. \\
& + A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin^2 \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin \phi \cos \phi \right] \\
& - A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin \phi \cos \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \sin^2 \phi \right] \\
& + A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) \sin \phi \cos \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \cos^2 \phi \right] \\
& + \left(\frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\rho + \frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\phi - \frac{\sin^3 \phi}{\rho} A_\phi B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\phi B_\phi \right) \\
& - \left(\frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\rho + \frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\phi + \frac{\sin \phi \cos^2 \phi}{\rho} A_\phi B_\rho + \frac{\cos^3 \phi}{\rho} A_\phi B_\phi \right) \\
& \left. + A_z \left(\frac{\partial B_\rho}{\partial z} \cos \phi - \frac{\partial B_\phi}{\partial z} \sin \phi \right) \right\} \hat{\mathbf{x}} \\
& + \left\{ A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \cos^2 \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin \phi \cos \phi \right] \right. \\
& + A_\rho \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin^2 \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin \phi \cos \phi \right] \\
& - A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin \phi \cos \phi - \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \sin^2 \phi \right] \\
& + A_\phi \left[\left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) \sin \phi \cos \phi + \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \cos^2 \phi \right] \\
& - \left(\frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\phi - \frac{\sin^2 \phi \cos \phi}{\rho} A_\phi B_\rho + \frac{\sin^3 \phi}{\rho} A_\phi B_\phi \right) \\
& + \left(\frac{\sin \phi \cos^2 \phi}{\rho} A_\rho B_\rho - \frac{\sin^2 \phi \cos \phi}{\rho} A_\rho B_\phi + \frac{\cos^3 \phi}{\rho} A_\phi B_\rho - \frac{\sin \phi \cos^2 \phi}{\rho} A_\phi B_\phi \right) \\
& \left. + A_z \left(\frac{\partial B_\rho}{\partial z} \sin \phi + \frac{\partial B_\phi}{\partial z} \cos \phi \right) \right\} \hat{\mathbf{y}} \\
& + \left[A_\rho \left(\frac{\partial B_z}{\partial \rho} \sin^2 \phi + \frac{\partial B_z}{\rho \partial \phi} \sin \phi \cos \phi + \frac{\partial B_z}{\partial \rho} \cos^2 \phi - \frac{\partial B_z}{\rho \partial \phi} \sin \phi \cos \phi \right) \right. \\
& \left. + A_\phi \left(\frac{\partial B_z}{\partial \rho} \sin \phi \cos \phi + \frac{\partial B_z}{\rho \partial \phi} \cos^2 \phi - \frac{\partial B_z}{\partial \rho} \sin \phi \cos \phi + \frac{\partial B_z}{\rho \partial \phi} \sin^2 \phi \right) + A_z \frac{\partial B_z}{\partial z} \right] \hat{\mathbf{z}}.
\end{aligned}$$

$$\begin{aligned}
(\mathbf{A} \cdot \nabla) \mathbf{B} = & \left[A_\rho \left(\frac{\partial B_\rho}{\partial \rho} \cos \phi - \frac{\partial B_\phi}{\partial \rho} \sin \phi \right) + A_\phi \left(\frac{\partial B_\rho}{\rho \partial \phi} \cos \phi - \frac{\partial B_\phi}{\rho \partial \phi} \sin \phi \right) \right. \\
& \left. - \left(\frac{\sin \phi}{\rho} A_\phi B_\rho + \frac{\cos \phi}{\rho} A_\phi B_\phi \right) + A_z \left(\frac{\partial B_\rho}{\partial z} \cos \phi - \frac{\partial B_\phi}{\partial z} \sin \phi \right) \right] \hat{\mathbf{x}} \\
& + \left[A_\rho \left(\frac{\partial B_\rho}{\partial \rho} \sin \phi + \frac{\partial B_\phi}{\partial \rho} \cos \phi \right) + A_\phi \left(\frac{\partial B_\rho}{\rho \partial \phi} \sin \phi + \frac{\partial B_\phi}{\rho \partial \phi} \cos \phi \right) \right.
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{\cos \phi}{\rho} A_\phi B_\rho - \frac{\sin \phi}{\rho} A_\phi B_\phi \right) + A_z \left(\frac{\partial B_\rho}{\partial z} \sin \phi + \frac{\partial B_\phi}{\partial z} \cos \phi \right) \Big] \hat{\mathbf{y}} \\
& + \left(A_\rho \frac{\partial B_z}{\partial \rho} + A_\phi \frac{\partial B_z}{\rho \partial \phi} + A_z \frac{\partial B_z}{\partial z} \right) \hat{\mathbf{z}}.
\end{aligned}$$

$$\begin{aligned}
(\mathbf{A} \cdot \nabla) \mathbf{B} &= A_\rho \left(\frac{\partial B_\rho}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{\partial B_\phi}{\partial \rho} \hat{\boldsymbol{\phi}} \right) + A_\phi \left(\frac{\partial B_\rho}{\rho \partial \phi} \hat{\boldsymbol{\rho}} + \frac{\partial B_\phi}{\rho \partial \phi} \hat{\boldsymbol{\phi}} \right) + A_z \left(\frac{\partial B_\rho}{\partial z} \hat{\boldsymbol{\rho}} + \frac{\partial B_\phi}{\partial z} \hat{\boldsymbol{\phi}} \right) \\
& - \rho^{-1} (A_\phi B_\phi \hat{\boldsymbol{\rho}} - A_\phi B_\rho \hat{\boldsymbol{\phi}}) + \left(A_\rho \frac{\partial B_z}{\partial \rho} + A_\phi \frac{\partial B_z}{\rho \partial \phi} + A_z \frac{\partial B_z}{\partial z} \right) \hat{\mathbf{z}}.
\end{aligned}$$

$$\begin{aligned}
(\mathbf{A} \cdot \nabla) \mathbf{B} &= A_\rho \left(\frac{\partial B_\rho}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{\partial B_\phi}{\partial \rho} \hat{\boldsymbol{\phi}} + \frac{\partial B_z}{\partial \rho} \hat{\mathbf{z}} \right) + A_\phi \rho^{-1} \left[\left(\frac{\partial B_\rho}{\partial \phi} - B_\phi \right) \hat{\boldsymbol{\rho}} + \left(\frac{\partial B_\phi}{\partial \phi} + B_\rho \right) \hat{\boldsymbol{\phi}} + \frac{\partial B_z}{\partial \phi} \hat{\mathbf{z}} \right] \\
& + A_z \left(\frac{\partial B_\rho}{\partial z} \hat{\boldsymbol{\rho}} + \frac{\partial B_\phi}{\partial z} \hat{\boldsymbol{\phi}} + \frac{\partial B_z}{\partial z} \hat{\mathbf{z}} \right).
\end{aligned}$$
