Problem 12-18) From Problem 34, part (b), we have $\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{B})=\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A})-\boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$. Replacing $\boldsymbol{B}$, which, in general, is a function of position $\boldsymbol{r}$, with the constant vector $\boldsymbol{C}$, makes the second curl on the right-hand side of the above identity to vanish. We will then have $\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{C})=\boldsymbol{C}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A})$. Applying Gauss's theorem to the left-hand-side of this equation, we find

$$\int_{\text{volume}}\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{C})\,\mathrm{d}v=\oint_{\text{surface}}[\boldsymbol{A}(\boldsymbol{r})\times\boldsymbol{C}]\cdot\mathrm{d}\boldsymbol{s}=-\oint_{\text{surface}}[\boldsymbol{A}(\boldsymbol{r})\times\mathrm{d}\boldsymbol{s}]\cdot\boldsymbol{C}=-\boldsymbol{C}\cdot\oint_{\text{surface}}\boldsymbol{A}(\boldsymbol{r})\times\mathrm{d}\boldsymbol{s}. \quad (1)$$

On the right-hand-side, the volume integral may be somewhat simplified, as follows:

$$\int_{\text{volume}}\boldsymbol{C}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A})\mathrm{d}\boldsymbol{v}=\boldsymbol{C}\cdot\int_{\text{volume}}\boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r})\,\mathrm{d}\boldsymbol{v}. \quad (2)$$

The above expressions are valid for *any* (arbitrary) constant vector $\boldsymbol{C}$ and, moreover, they are equal to each other. We conclude that the coefficients of $\boldsymbol{C}$ on the right-hand-sides of Eqs. (1) and (2) must be the same, that is, $\int_{\text{volume}}\boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r})\,\mathrm{d}\boldsymbol{v}=-\oint_{\text{surface}}\boldsymbol{A}(\boldsymbol{r})\times\mathrm{d}\boldsymbol{s}$.