Problem 12-17) From Problem 34, part (a), we have $\boldsymbol{\nabla}\cdot(\psi\boldsymbol{A})=(\boldsymbol{\nabla}\psi)\cdot\boldsymbol{A}+\psi\boldsymbol{\nabla}\cdot\boldsymbol{A}$. Replacing $\boldsymbol{A}$, which, in general, is a function of position $\boldsymbol{r}$, with the constant vector $\boldsymbol{C}$, makes the divergence term on the right-hand side of the above identity to vanish. We will then have $\boldsymbol{\nabla}\cdot(\psi\boldsymbol{C})=(\boldsymbol{\nabla}\psi)\cdot\boldsymbol{C}$. Applying Gauss's theorem to the left-hand-side of the above equation, we find

$$\int_{\text{volume}}\boldsymbol{\nabla}\cdot(\psi\,\boldsymbol{C})\,\mathrm{d}v=\oint_{\text{surface}}\psi(\boldsymbol{r})\,\boldsymbol{C}\cdot\mathrm{d}\boldsymbol{s}=\boldsymbol{C}\cdot\oint_{\text{surface}}\psi(\boldsymbol{r})\mathrm{d}\boldsymbol{s}. \tag{1}$$

On the right-hand-side, the volume integral may be somewhat simplified, as follows:

$$\int_{\text{volume}}(\boldsymbol{\nabla}\psi)\cdot\boldsymbol{C}\,\mathrm{d}\boldsymbol{v}=\boldsymbol{C}\cdot\int_{\text{volume}}\boldsymbol{\nabla}\psi(\boldsymbol{r})\,\mathrm{d}\boldsymbol{v}. \tag{2}$$

The above expressions are valid for *any* (arbitrary) constant vector $\boldsymbol{C}$ and, moreover, they are equal to each other. We conclude that the coefficients of $\boldsymbol{C}$ on the right-hand-sides of Eqs. (1) and (2) must be the same, that is, $\int_{\text{volume}}\boldsymbol{\nabla}\psi(\boldsymbol{r})\,\mathrm{d}\boldsymbol{v}=\oint_{\text{surface}}\psi(\boldsymbol{r})\mathrm{d}\boldsymbol{s}$.