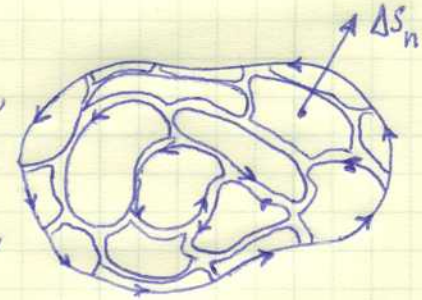
Problem 16)

Break up the surface S into a collection of small surfaces, $\Delta S_1, \Delta S_2, \ldots,$ with shared boundaries that cover the entire surface S. For each such small surface we have,



$$\int_{\Delta S_n} \vec{B}\cdot d\vec{s} \cong (\vec{\nabla}\times\vec{A})\cdot\Delta\vec{S}_n = \frac{\oint \vec{A}\cdot d\vec{\ell}}{\Delta S_n}\,\Delta S_n = \oint_{\left(\substack{\text{Boundary}\\ \text{of } \Delta S_n}\right)} \vec{A}\cdot d\vec{\ell}$$

When the contributions of all the small surfaces are added up, the line-integrals on adjacent (shared) boundaries will cancel out.

What is left is the line-integral around the closed loop C that bounds the surface S. Therefore,

$$\int_S \vec{B}\cdot d\vec{s} \cong \sum_n \int_{\Delta S_n} \vec{B}\cdot d\vec{s} = \oint_C \vec{A}\cdot d\vec{\ell} \quad \leftarrow \text{stokes's theorem}$$

When the surface S is closed, its boundary C shrinks to zero. Then $\oint \vec{A}\cdot d\vec{\ell} = 0 \Rightarrow \oint_{\left(\substack{\text{Closed}\\ \text{Surface}}\right)} \vec{B}\cdot d\vec{s} = 0$. When the volume enclosed becomes small, $\vec{\nabla}\cdot\vec{B} = 0$.