Opti 503A **Solutions**

Problem 14)

a) $\vec{\nabla}\cdot(\psi\vec{A}) = \frac{\partial}{\partial x}(\psi A_x) + \frac{\partial}{\partial y}(\psi A_y) + \frac{\partial}{\partial z}(\psi A_z) =$

$\left(A_x \frac{\partial \psi}{\partial x} + \psi \frac{\partial A_x}{\partial x}\right) + \left(A_y \frac{\partial \psi}{\partial y} + \psi \frac{\partial A_y}{\partial y}\right) + \left(A_z \frac{\partial \psi}{\partial z} + \psi \frac{\partial A_z}{\partial z}\right) =$

$\left(A_x \frac{\partial \psi}{\partial x} + A_y \frac{\partial \psi}{\partial y} + A_z \frac{\partial \psi}{\partial z}\right) + \psi\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) =$

$\vec{A}\cdot(\vec{\nabla}\psi) + \psi(\vec{\nabla}\cdot\vec{A})$ ✓

b) $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \frac{\partial}{\partial x}(A_y B_z - A_z B_y) + \frac{\partial}{\partial y}(A_z B_x - A_x B_z)$

$+ \frac{\partial}{\partial z}(A_x B_y - A_y B_x) = \left(B_z \frac{\partial A_y}{\partial x} + A_y \frac{\partial B_z}{\partial x} - B_y \frac{\partial A_z}{\partial x} - A_z \frac{\partial B_y}{\partial x}\right) +$

$\left(B_x \frac{\partial A_z}{\partial y} + A_z \frac{\partial B_x}{\partial y} - B_z \frac{\partial A_x}{\partial y} - A_x \frac{\partial B_z}{\partial y}\right) + \left(B_y \frac{\partial A_x}{\partial z} + A_x \frac{\partial B_y}{\partial z} - B_x \frac{\partial A_y}{\partial z}\right.$

$\left. - A_y \frac{\partial B_x}{\partial z}\right) = B_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + B_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + B_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)$

$+ A_z\left(\frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x}\right) + A_y\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right) + A_x\left(\frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y}\right) =$

$\vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$ ✓