Opti 503A **Solutions**

Problem 13)

In spherical coordinates, the vector-field $\vec{A}(\vec{r}) = \vec{r}$ has components $A_r = r$, $A_\theta = A_\phi = 0$. Therefore,

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^3) = \frac{1}{r^2}(3r^2) = 3.$$

Thus $\vec{\nabla}\cdot\vec{r} = 3$

$$\vec{\nabla}\cdot\vec{n} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2) = \frac{2}{r}$$

$$\vec{\nabla}\times\vec{r} = \vec{\nabla}\times(x\hat{x} + y\hat{y} + z\hat{z}) = \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right)\hat{x} + \left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x}\right)\hat{y}$$

$$+\left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial z}\right)\hat{z} \Rightarrow \vec{\nabla}\times\vec{r} = 0$$

$$\vec{\nabla}\times\vec{n} = \vec{\nabla}\times\left(\frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2+y^2+z^2}}\right) = \left[\frac{\partial}{\partial y}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) - \frac{\partial}{\partial z}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)\right]\hat{x}$$

$$+[\cdots]\hat{y} + [\cdots]\hat{z} = \left[\frac{-yz}{(x^2+y^2+z^2)^{3/2}} + \frac{yz}{(x^2+y^2+z^2)^{3/2}}\right]\hat{x}$$

$$+[\cdots]\hat{y} + [\cdots]\hat{z} = 0 \Longrightarrow \vec{\nabla}\times\vec{n} = 0$$