Problem 12) In Cartesian coordinates we have $\boldsymbol{C}\cdot\Delta\boldsymbol{S} = C_x\Delta S_x + C_y\Delta S_y + C_z\Delta S_z$. Now, by definition of the curl operator, $C_x\Delta S_x$ is the integral of $\boldsymbol{A}(\boldsymbol{r})$ around the triangle in the yz-plane. Similarly, $C_y\Delta S_y$ is the integral of $\boldsymbol{A}(\boldsymbol{r})$ around the triangle in the xz-plane, while $C_z\Delta S_z$ is the integral of $\boldsymbol{A}(\boldsymbol{r})$ around the triangle in the xy-plane. When these three integrals are added together, the contributions to the integrals by those sides of the triangles which are shared between adjacent triangles (i.e., the sides of the triangles on the x, y, and z axes) cancel out. What is left then is the contribution by the sides of the triangle at the base of the pyramid. Therefore, $\boldsymbol{C}\cdot\Delta\boldsymbol{S}$ is the integral of $\boldsymbol{A}(\boldsymbol{r})$ around the base of the pyramid. Note that, for the contributions of adjacent sides to cancel out, the right-hand rule must be observed for each triangle.

This exercise confirms that the curl at any given point need only be evaluated along three orthogonal directions. Once C_x, C_y, C_z are known, the loop integral of $\boldsymbol{A}(\boldsymbol{r})$ around any other triangle (e.g., the base triangle) can be obtained from the dot-product $\boldsymbol{C}\cdot\Delta\boldsymbol{S}$.