

**Problem 12-8)** Method 1: Define  $\mathbf{C} = \mathbf{B} \times \mathbf{A}$ . Then from Problem 3, we will have

$$[\mathbf{A} \times (\mathbf{B} \times \mathbf{A})] \cdot \mathbf{B} = (\mathbf{A} \times \mathbf{C}) \cdot \mathbf{B} = \mathbf{C} \cdot (\mathbf{B} \times \mathbf{A}) = \mathbf{C} \cdot (-\mathbf{A} \times \mathbf{B}) = -\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) = (\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{A} \times \mathbf{B}).$$

Method 2: Use Problem 5 to write

$$\begin{aligned} [\mathbf{A} \times (\mathbf{B} \times \mathbf{A})] \cdot \mathbf{B} &= [(\mathbf{A} \cdot \mathbf{A})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{A}] \cdot \mathbf{B} = (\mathbf{A} \cdot \mathbf{A})(\mathbf{B} \cdot \mathbf{B}) - (\mathbf{A} \cdot \mathbf{B})(\mathbf{A} \cdot \mathbf{B}) \\ &= |\mathbf{A}|^2 |\mathbf{B}|^2 - |\mathbf{A}|^2 |\mathbf{B}|^2 \cos^2 \theta = |\mathbf{A}|^2 |\mathbf{B}|^2 \sin^2 \theta = |\mathbf{A} \times \mathbf{B}|^2 = (\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{A} \times \mathbf{B}). \end{aligned}$$

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