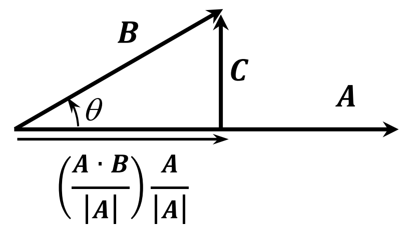
Problem 12-6) Consider the projection of $\boldsymbol{B}$ onto the unit-vector $\boldsymbol{A}/|\boldsymbol{A}|$, that is,



$$\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}|}=\frac{|\boldsymbol{A}|\,|\boldsymbol{B}|\cos\theta}{|\boldsymbol{A}|}=|\boldsymbol{B}|\cos\theta.$$

Since the unit-vector $\boldsymbol{A}/|\boldsymbol{A}|$ is aligned with $\boldsymbol{A}$, the vector $\left(\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}|}\right)\frac{\boldsymbol{A}}{|\boldsymbol{A}|}$ has length $|\boldsymbol{B}|\cos\theta$ and is parallel to $\boldsymbol{A}$; in other words, it is the "shadow" of $\boldsymbol{B}$ on $\boldsymbol{A}$. When this shadow projection is removed (i.e., subtracted) from $\boldsymbol{B}$, what remains is $\boldsymbol{C}$, which is perpendicular to $\boldsymbol{A}$. This can be shown directly, as follows:

$$\boldsymbol{A}\cdot\boldsymbol{C}=\boldsymbol{A}\cdot\left(\boldsymbol{B}-\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}|^2}\,\boldsymbol{A}\right)=(\boldsymbol{A}\cdot\boldsymbol{B})-\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}|^2}(\boldsymbol{A}\cdot\boldsymbol{A}).$$

But $\boldsymbol{A}\cdot\boldsymbol{A}=|\boldsymbol{A}|^2$; therefore, $\boldsymbol{A}\cdot\boldsymbol{C}=\boldsymbol{A}\cdot\boldsymbol{B}-\boldsymbol{A}\cdot\boldsymbol{B}=0$, which implies that the angle between $\boldsymbol{A}$ and $\boldsymbol{C}$ is 90°, that is, $\boldsymbol{A}$ and $\boldsymbol{C}$ are orthogonal to each other.