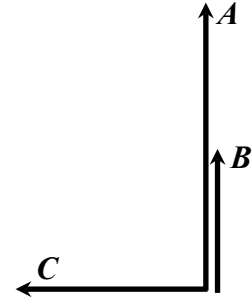


**Problem 12-5)** a) Consider the three vectors shown on the right-hand side. Since  $\mathbf{A}$  and  $\mathbf{B}$  are parallel,  $\mathbf{A} \times \mathbf{B} = 0$ . Therefore,  $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = 0$ . In contrast,  $\mathbf{B} \times \mathbf{C}$  has magnitude  $BC = |\mathbf{B}||\mathbf{C}|$  and is perpendicular to the plane of the paper. Cross-multiplying  $\mathbf{A}$  into  $\mathbf{B} \times \mathbf{C}$  thus yields a vector of magnitude  $ABC = |\mathbf{A}||\mathbf{B}||\mathbf{C}|$ , which is anti-parallel with  $\mathbf{C}$ . Therefore,  $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$  is not necessarily equal to  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ .



$$\begin{aligned}
 \text{b) } \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) \times [(B_y C_z - B_z C_y) \hat{x} + (B_z C_x - B_x C_z) \hat{y} + (B_x C_y - B_y C_x) \hat{z}] \\
 &= [A_y (B_x C_y - B_y C_x) - A_z (B_z C_x - B_x C_z)] \hat{x} \\
 &\quad + [A_z (B_y C_z - B_z C_y) - A_x (B_x C_y - B_y C_x)] \hat{y} \\
 &\quad + [A_x (B_z C_x - B_x C_z) - A_y (B_y C_z - B_z C_y)] \hat{z} \\
 &= (A_y C_y + A_z C_z) B_x \hat{x} - (A_y B_y + A_z B_z) C_x \hat{x} \\
 &\quad + (A_x C_x + A_z C_z) B_y \hat{y} - (A_x B_x + A_z B_z) C_y \hat{y} \\
 &\quad + (A_x C_x + A_y C_y) B_z \hat{z} - (A_x B_x + A_y B_y) C_z \hat{z} \\
 &= (A_x C_x + A_y C_y + A_z C_z) B_x \hat{x} - (A_x B_x + A_y B_y + A_z B_z) C_x \hat{x} \\
 &\quad + (A_x C_x + A_y C_y + A_z C_z) B_y \hat{y} - (A_x B_x + A_y B_y + A_z B_z) C_y \hat{y} \\
 &\quad + (A_x C_x + A_y C_y + A_z C_z) B_z \hat{z} - (A_x B_x + A_y B_y + A_z B_z) C_z \hat{z} \\
 &= (\mathbf{A} \cdot \mathbf{C})(B_x \hat{x} + B_y \hat{y} + B_z \hat{z}) - (\mathbf{A} \cdot \mathbf{B})(C_x \hat{x} + C_y \hat{y} + C_z \hat{z}) \\
 &= (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}.
 \end{aligned}$$

**Geometric Interpretation:** The vector  $\mathbf{B} \times \mathbf{C}$  is perpendicular to the plane of  $\mathbf{B}$  and  $\mathbf{C}$ . Therefore, when cross-multiplied into  $\mathbf{A}$ , the resulting vector must lie in the plane of  $\mathbf{B}$  and  $\mathbf{C}$ . In other words, the vector  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$  must be co-planar with  $\mathbf{B}$  and  $\mathbf{C}$ . As such, it can be written as a linear combination of  $\mathbf{B}$  and  $\mathbf{C}$ , that is,  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \alpha \mathbf{B} + \beta \mathbf{C}$ . This is all that one can say about the  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$  triple cross-product. The fact that  $\alpha = \mathbf{A} \cdot \mathbf{C}$  and  $\beta = -\mathbf{A} \cdot \mathbf{B}$  comes out of the preceding calculations, with no obvious geometric interpretation.