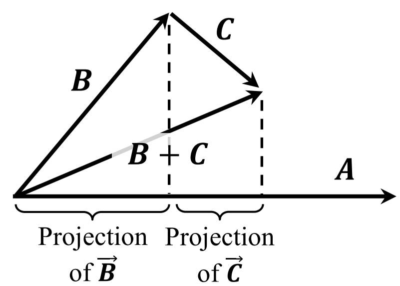
Problem 12-4)

a) $\boldsymbol{A} \cdot \boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}| \cos\theta = \boldsymbol{B} \cdot \boldsymbol{A}$.

b)
$$\begin{aligned}
\boldsymbol{A} \cdot (\boldsymbol{B} + \boldsymbol{C}) &= (A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}) \cdot [(B_x + C_x)\hat{\boldsymbol{x}} + (B_y + C_y)\hat{\boldsymbol{y}} + (B_z + C_z)\hat{\boldsymbol{z}}] \\
&= A_x(B_x + C_x) + A_y(B_y + C_y) + A_z(B_z + C_z) \\
&= (A_xB_x + A_yB_y + A_zB_z) + (A_xC_x + A_yC_y + A_zC_z) \\
&= \boldsymbol{A} \cdot \boldsymbol{B} + \boldsymbol{A} \cdot \boldsymbol{C}.
\end{aligned}$$

Geometric interpretation: The projection of $\boldsymbol{B} + \boldsymbol{C}$ on $\boldsymbol{A}$ has the same length as the projection of $\boldsymbol{B}$ on $\boldsymbol{A}$, plus that of $\boldsymbol{C}$ on $\boldsymbol{A}$. (Note: The vectors $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ in the diagram are not necessarily in the same plane.)



c) $\boldsymbol{A} \times \boldsymbol{B}$ and $\boldsymbol{B} \times \boldsymbol{A}$ are both perpendicular to the plane defined by $\boldsymbol{A}$ and $\boldsymbol{B}$, and both have a magnitude equal to the area of the parallelogram formed by $\boldsymbol{A}$ and $\boldsymbol{B}$. The right-hand rule, however, gives opposite directions to $\boldsymbol{A} \times \boldsymbol{B}$ and $\boldsymbol{B} \times \boldsymbol{A}$. Therefore, $\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$.

d)
$$\begin{aligned}
\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{C}) &= (A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}) \times [(B_x + C_x)\hat{\boldsymbol{x}} + (B_y + C_y)\hat{\boldsymbol{y}} + (B_z + C_z)\hat{\boldsymbol{z}}] \\
&= [A_y(B_z + C_z) - A_z(B_y + C_y)]\hat{\boldsymbol{x}} + [A_z(B_x + C_x) - A_x(B_z + C_z)]\hat{\boldsymbol{y}} \\
&\quad + [A_x(B_y + C_y) - A_y(B_x + C_x)]\hat{\boldsymbol{z}} \\
&= (A_yB_z - A_zB_y)\hat{\boldsymbol{x}} + (A_yC_z - A_zC_y)\hat{\boldsymbol{x}} \\
&\quad + (A_zB_x - A_xB_z)\hat{\boldsymbol{y}} + (A_zC_x - A_xC_z)\hat{\boldsymbol{y}} \\
&\quad + (A_xB_y - A_yB_x)\hat{\boldsymbol{z}} + (A_xC_y - A_yC_x)\hat{\boldsymbol{z}} \\
&= \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{C}.
\end{aligned}$$

e) $(\boldsymbol{A} + \boldsymbol{B}) \times \boldsymbol{C} = -\boldsymbol{C} \times (\boldsymbol{A} + \boldsymbol{B}) = -\boldsymbol{C} \times \boldsymbol{A} - \boldsymbol{C} \times \boldsymbol{B} = \boldsymbol{A} \times \boldsymbol{C} + \boldsymbol{B} \times \boldsymbol{C}$.