Solution to Problem 15) To raise a diagonalizable matrix $A = \tilde{V}\Lambda\tilde{V}^{-1}$ to the power of the rational number m/n (preferably, with the ratio reduced, so that m and n have no common factors), one must first find all the matrices $A^{1/n} = \tilde{V}\Lambda^{1/n}\tilde{V}^{-1}$, where the matrix $\Lambda^{1/n}$ is a diagonal matrix whose diagonal elements are $\lambda_k^{1/n}$. Here λ_k is the k^{th} eigen-value of A. (Assuming that A has $\nu \le N$ nonzero eigen-values, the total number of matrices $A^{1/n}$ will be n^ν.) Afterward, each matrix $A^{1/n}$ must be raised to the power of m in order to arrive at one of the matrices $A^{m/n}$.

For the general case of A^α, where α could be any real or complex number, we now define $A^\alpha = \tilde{V}\Lambda^\alpha\tilde{V}^{-1}$, with the diagonal elements of Λ^α being $\lambda_k^\alpha = \exp(\alpha \ln \lambda_k)$. Recalling that $\ln \lambda_k = \ln|\lambda_k| + \mathrm{i}(\varphi_k + 2\mu\pi)$, where μ is any arbitrary integer (positive, zero, or negative), it is clear that, depending on the value of α, the matrices A^α could be finite or infinite in number.