

Solution to Problem 10) The eigen-values of A are solutions of its characteristic equation $|A - \lambda I| = 0$. If at least one solution of this N^{th} order polynomial equation happens to be zero, say, $\lambda_n = 0$, we will have $|A - \lambda_n I| = |A| = 0$. Thus, the matrix A , whose determinant is found to be zero, cannot be inverted.

If A is diagonalizable, its determinant will be equal to the product $\lambda_1 \lambda_2 \cdots \lambda_N$ of its eigen-values. Thus, a single eigen-value of zero magnitude is sufficient to ensure that $|A| = 0$, which is the necessary and sufficient condition for A to be non-invertible.
