

**Solution to Problem 3)** By definition, the inverse  $(AB)^{-1}$  of the product matrix  $AB$  must have the following properties: (i)  $(AB)^{-1}(AB) = I$ , and (ii)  $(AB)(AB)^{-1} = I$ . Below, we verify that the product matrix  $B^{-1}A^{-1}$  does, in fact, satisfy both requirements.

$$\text{i) } (B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = B^{-1}B = I.$$

$$\text{ii) } (AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I.$$

Consequently,  $B^{-1}A^{-1}$  is the desired inverse  $(AB)^{-1}$  of the matrix  $AB$ .

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