Solution to Problem 2) A general 3×3 matrix may be written as follows:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

The matrix C of the co-factors of A is then given by

$$C = \begin{pmatrix} a_{22}a_{33} - a_{23}a_{32} & a_{23}a_{31} - a_{21}a_{33} & a_{21}a_{32} - a_{22}a_{31} \\ a_{13}a_{32} - a_{12}a_{33} & a_{11}a_{33} - a_{13}a_{31} & a_{12}a_{31} - a_{11}a_{32} \\ a_{12}a_{23} - a_{13}a_{22} & a_{13}a_{21} - a_{11}a_{23} & a_{11}a_{22} - a_{12}a_{21} \end{pmatrix}.$$

Taking the first row of A and the first row of C, then multiplying their corresponding elements together and adding them up, we find the determinant of A, as follows:

$$|A| = a_{11}\underline{\underline{a_{22}}}a_{33} - a_{11}\underline{\underline{a_{23}}}a_{32} + a_{12}\underline{\underline{a_{23}}}a_{31} - a_{12}\underline{\underline{a_{21}}}a_{33} + a_{13}\underline{\underline{a_{21}}}a_{32} - a_{13}\underline{\underline{a_{22}}}a_{31}.$$

The process may be repeated for any row (or any column) of A, provided that the same row (and the same column) is chosen from the matrix C. For instance, element-by-element multiplication of the third columns of A and C, followed by summation, yields

$$|A| = a_{13}\underline{\underline{a_{21}}}a_{32} - a_{13}\underline{\underline{a_{22}}}a_{31} + a_{23}\underline{\underline{a_{12}}}a_{31} - a_{23}\underline{\underline{a_{11}}}a_{32} + a_{33}\underline{\underline{a_{11}}}a_{22} - a_{33}\underline{\underline{a_{12}}}a_{21}.$$

To check Property 2 of the co-factors (as listed in Sec.4), we pick the first row from A and the third row from C. The sum over the element-by-element product of these two rows will be

$$a_{11}a_{12}a_{23} - a_{11}a_{13}a_{22} + a_{12}a_{13}a_{21} - a_{12}a_{11}a_{23} + a_{13}a_{11}a_{22} - a_{13}a_{12}a_{21} = 0.$$

A similar operation performed on any pair of rows (or any pair of columns) would yield the same null result.

As for Property 3, note that transposing the matrix A would cause the indices m and n of a_{mn} to switch places. A quick glance at the elements of the matrix C above reveals that switching the indices m and n would turn C into its transpose. Therefore, the matrix of the co-factors of A^T is the transpose C^T of the matrix C of the co-factors of A.