Solution to Problem 1) When the product AB is calculated at first, we will have

$$(AB)_{km} = \sum_{l=1}^{L} A_{kl} B_{lm}. \tag{1}$$

Subsequent multiplication of AB by C on the right-hand-side then yields

$$(ABC)_{kn} = \sum_{m=1}^{M} (AB)_{km} C_{mn} = \sum_{m=1}^{M} \sum_{l=1}^{L} (A_{kl} B_{lm} C_{mn}). \tag{2}$$

In contrast, when the product BC is computed at first, we find

$$(BC)_{ln} = \sum_{m=1}^{M} B_{lm} C_{mn}. \tag{3}$$

Subsequent multiplication of BC by A on the left-hand-side then yields

$$(ABC)_{kn} = \sum_{l=1}^{L} A_{kl} \, (BC)_{ln} = \sum_{l=1}^{L} \sum_{m=1}^{M} (A_{kl} B_{lm} C_{mn}). \tag{4}$$

Clearly, the two double-sums appearing in Eqs.(2) and (4) are identical, as the order in which the triple-products $A_{kl} B_{lm} C_{mn}$ are added together is irrelevant. (The indices k and n are fixed, and the double-sum is over all possible pairs of indices l and m. It does not matter whether m is fixed initially while l is being varied, or l is fixed initially while m is being varied. So long as all possible combinations of l and m are included in the double-sum, the end result will be the same.)