Solution to Problem 6) Euler's reflection formula $\Gamma(z)\Gamma(1-z) = \pi/\sin(\pi z)$, and the functional relation $\Gamma(z+1) = z\Gamma(z)$, will be needed for solving this problem, as follows:

$$\Gamma(1+x+\mathrm{i}y)\Gamma(1-x-\mathrm{i}y) = (x+\mathrm{i}y)\Gamma(x+\mathrm{i}y)\Gamma(1-x-\mathrm{i}y) = \frac{\pi(x+\mathrm{i}y)}{\sin[\pi(x+\mathrm{i}y)]}.$$

$$\Gamma(1+x-\mathrm{i}y)\Gamma(1-x+\mathrm{i}y) = (x-\mathrm{i}y)\Gamma(x-\mathrm{i}y)\Gamma(1-x+\mathrm{i}y) = \frac{\pi(x-\mathrm{i}y)}{\sin[\pi(x-\mathrm{i}y)]}.$$

Multiplication of the above equations now yields

$$\Gamma(1+x+\mathrm{i}y)\Gamma(1-x+\mathrm{i}y)\Gamma(1+x-\mathrm{i}y)\Gamma(1-x-\mathrm{i}y) = \frac{\pi^2(x^2+y^2)}{\sin[\pi(x+\mathrm{i}y)]\sin[\pi(x-\mathrm{i}y)]}$$

$$= \frac{(2\mathrm{i}\pi)^2(x^2+y^2)}{\{\exp[\mathrm{i}\pi(x+\mathrm{i}y)] - \exp[-\mathrm{i}\pi(x+\mathrm{i}y)]\} \times \{\exp[\mathrm{i}\pi(x-\mathrm{i}y)] - \exp[-\mathrm{i}\pi(x-\mathrm{i}y)]\}}$$

$$= -\frac{4\pi^2(x^2+y^2)}{\exp(2\mathrm{i}\pi x) + \exp(-2\mathrm{i}\pi x) - \exp(-2\pi y) - \exp(2\pi y)} = \frac{2\pi^2(x^2+y^2)}{\cosh(2\pi y) - \cos(2\pi x)}.$$
