Solution to Problem 2) The general formulas for the forward and inverse Fourier transform of the function $f(x)$ are

$$F(s) = \int_{-\infty}^{\infty} f(x)\exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x \quad \rightarrow \quad f(x) = \int_{-\infty}^{\infty} F(s)\exp(\mathrm{i}2\pi sx)\,\mathrm{d}s.$$

From these equations, using straightforward differentiation, we obtain

$$f''(x) = \int_{-\infty}^{\infty} (\mathrm{i}2\pi s)^2 F(s)\exp(\mathrm{i}2\pi sx)\,\mathrm{d}s.$$

$$F'(s) = -\mathrm{i}2\pi \int_{-\infty}^{\infty} xf(x)\exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x.$$

$$xf(x) = -(\mathrm{i}2\pi)^{-1} \int_{-\infty}^{\infty} F'(s)\exp(\mathrm{i}2\pi sx)\,\mathrm{d}s.$$

Therefore, Airy's differential equation may be Fourier transformed as follows:

$$f''(x) - xf(x) = (\mathrm{i}2\pi)^{-1} \int_{-\infty}^{\infty} [F'(s) + (\mathrm{i}2\pi)^3 s^2 F(s)]\exp(\mathrm{i}2\pi sx)\,\mathrm{d}s = 0.$$

From the above equation, we conclude that $F'(s) + (\mathrm{i}2\pi)^3 s^2 F(s) = 0$, and, therefore,

$$F(s) = c\exp[-⅓(\mathrm{i}2\pi)^3 s^3] = c\exp[⅓\mathrm{i}(2\pi s)^3].$$ ← *c* is the integration constant

The above transformation of Airy's 2^{nd} order differential equation has produced a 1^{st} order differential equation in $F(s)$. The resulting solution thus applies to $\mathrm{Ai}(x)$ only, since the Airy function of the 2^{nd} kind, $\mathrm{Bi}(x)$, is divergent and, therefore, does not have a Fourier transform. Considering that the value of the Fourier transform $F(s)$ of $f(x)$ at $s = 0$ equals the area under $f(x)$, and that the area under $\mathrm{Ai}(x)$, namely, $\int_{-\infty}^{\infty} \mathrm{Ai}(x)\mathrm{d}x$, is equal to 1, the integration constant c in the above expression of $F(s)$ must be set to 1.