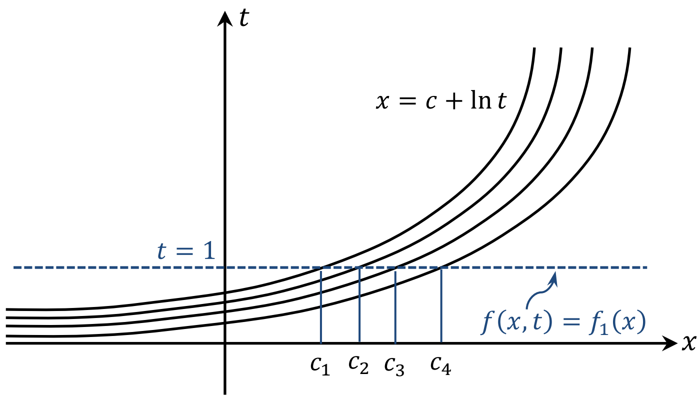
Problem 19) The characteristic base curves parameterized by p satisfy the equations $\mathrm{d}x/\mathrm{d}p = 1$ and $\mathrm{d}t/\mathrm{d}p = t$. Consequently, $\mathrm{d}x = \mathrm{d}t/t$, which, upon integration, yields $x = c + \ln t$, where c is the integration constant. Denoting the position in the xt-plane by $\boldsymbol{r} = x\hat{\boldsymbol{x}} + t\hat{\boldsymbol{t}}$, the differential equation becomes $\mathrm{d}\boldsymbol{r} \cdot \boldsymbol{\nabla} f(x,t)/\mathrm{d}p = 0$, which indicates that $f(x,t)$ remains constant along each and every characteristic base curve.



The initial condition specifies the value of the function at $t = 1$, for which the x-coordinate of the generic base curve is c and, therefore, the initial value of the function at $(x,t) = (c,1)$ equals $f_1(c)$. Since the function must remain constant along the base curve, we arrive at the following general solution (in the region $t > 0$) of the differential equation:

$$f(x,t) = f_1(x - \ln t).$$
