Problem 1) a)

$$\alpha\,\frac{\zeta(x+\Delta x,t)-\zeta(x,t)}{\Delta x}-\alpha\,\frac{\zeta(x,t)-\zeta(x-\Delta x,t)}{\Delta x}-\beta\Delta x\,\frac{\partial\zeta(x,t)}{\partial t}=\rho\Delta x\,\frac{\partial^2\zeta(x,t)}{\partial t^2}$$

$$\Rightarrow \alpha\left[\frac{\partial\zeta(x+\tfrac{1}{2}\Delta x,t)}{\partial x}-\frac{\partial\zeta(x-\frac{\Delta x}{2},t)}{\partial x}\right]=\rho\Delta x\,\frac{\partial^2\zeta(x,t)}{\partial t^2}+\beta\Delta x\,\frac{\partial\zeta(x,t)}{\partial t}$$

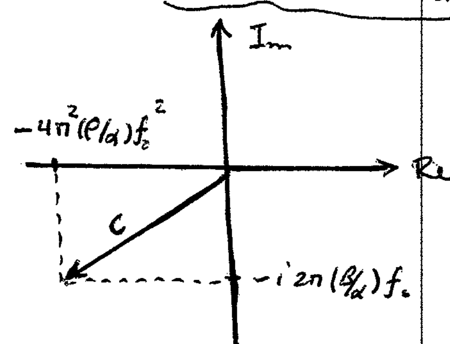
$$\Rightarrow (\alpha/\rho)\frac{\partial^2\zeta(x,t)}{\partial x^2}=\frac{\partial^2\zeta(x,t)}{\partial t^2}+\left(\frac{\beta}{\rho}\right)\frac{\partial\zeta(x,t)}{\partial t}$$

b) Separation of Variables: $\zeta(x,t)=f(x)\,g(t)\Rightarrow$

$$(\alpha/\rho)\,f''(x)\,g(t)=f(x)g''(t)+(\beta/\rho)f(x)g'(t)\Rightarrow\frac{f''(x)}{f(x)}=\frac{g''(t)+(\beta/\rho)g'(t)}{(\alpha/\rho)\,g(t)}=c$$

The time-dependence $g(t)$ is given by the excitation function as $g(t)=e^{-i2\pi f_0 t}$.

Therefore $$\frac{g''(t)+(\beta/\rho)g'(t)}{(\alpha/\rho)g(t)}=\frac{(-i2\pi f_0)^2+(\beta/\rho)(-i2\pi f_0)}{(\alpha/\rho)}=-4\pi^2(\rho/\alpha)f_0^2-i2\pi(\beta/\alpha)f_0.$$



The Complex Constant c shown in the figure has two roots. We pick for $\sqrt{c}$ that root which is in the upper half of the Complex plane.

As will be seen below, this root will cause the amplitude of the propagating wave to decay with increasing x, as the wave propagates from left to right along the wire. We'll have $f(x)=e^{\sqrt{c}\,x}$. Therefore, $\zeta(x,t)=\zeta_0\,e^{\sqrt{c}\,x}\,e^{-i2\pi f_0 t}$. Since the real part of $\sqrt{c}$ is negative, the wave decays with increasing x. Since the imaginary part of $\sqrt{c}$

is positive, the wave propagates from left to write as time t increases.

c) $\sqrt{c} = \left[-4\pi^2 (\rho/\alpha) f_0^2 - i\, 2\pi (\beta/\alpha) f_0\right]^{1/2} = i\, 2\pi f_0 \sqrt{\rho/\alpha}\, \sqrt{1 + i \frac{\beta}{2\pi f_0 \rho}}$

$$\simeq i\, 2\pi f_0 \sqrt{\rho/\alpha}\, \left(1 + i \frac{\beta}{4\pi f_0 \rho}\right) \qquad \sqrt{1+\epsilon} \simeq 1 + \frac{\epsilon}{2}$$

$$= i\, 2\pi f_0 \sqrt{\rho/\alpha} - \frac{\beta}{2\sqrt{\rho\alpha}}$$

The propagating wave will then become $\xi(x,t) \cong x_0\, e^{-\frac{\beta}{2\sqrt{\rho\alpha}} x}\, e^{i 2\pi f_0 (\sqrt{\rho/\alpha}\, x - t)}$

The phase velocity is thus seen to be $v = \sqrt{\alpha/\rho}$, while the attenuation coefficient is given by $\frac{\beta}{2\sqrt{\rho\alpha}}$. To see that the phase velocity is given by the above formula, note that the imaginary exponent, which represents the phase of the waveform, is $2\pi f_0 (\sqrt{\rho/\alpha}\, x - t)$. As the wave propagates, this phase must remain constant, that is, $\sqrt{\rho/\alpha}\, \Delta x - \Delta t = 0$. From this we find $v = \frac{\Delta x}{\Delta t} = \sqrt{\alpha/\rho}$.

d) Let $\underset{\sim}{\xi}_0(f) = \mathcal{F}\{\xi_0(t)\}$. We may write $\xi_0(t) = \int_{-\infty}^{\infty} \underset{\sim}{\xi}_0(f)\, e^{-i 2\pi f t}\, df$.

$$\Rightarrow \xi(x,t) = \int_{-\infty}^{\infty} \underset{\sim}{\xi}_0(f)\, e^{-\frac{\beta}{2\sqrt{\rho\alpha}} x}\, e^{i 2\pi f (\sqrt{\rho/\alpha}\, x - t)}\, df = e^{-\frac{\beta x}{2\sqrt{\rho\alpha}}}\, \xi_0(t - \sqrt{\rho/\alpha}\, x).$$

The attenuation is an exponentially-decreasing function of x, and the time delay is $\sqrt{\rho/\alpha}\, x = x/v$.