Problem 46) a) In the convolution operation, one of the functions is flipped around the vertical axis before being shifted and multiplied by the other function prior to integration. In the case of cross-correlation, neither $f(x)$ nor $g(x)$ is flipped; rather, one function is shifted along the x-axis then multiplied by the other function. Finally, the product is integrated along the entire x-axis.

b) $$\mathcal{F}\{f(x)\otimes g(x)\} = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(x')g(x'-x)\mathrm{d}x'\right]\exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x$$

reversing the order of integration → $$= \int_{-\infty}^{\infty} f(x')\left[\int_{-\infty}^{\infty} g(x'-x)\exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x\right]\mathrm{d}x'$$

change of variable: $y = x' - x$ → $$= \int_{-\infty}^{\infty} f(x')\left[\int_{-\infty}^{\infty} g(y)\exp[-\mathrm{i}2\pi s(x'-y)]\,\mathrm{d}y\right]\mathrm{d}x'$$

splitting the complex exponential → $$= \int_{-\infty}^{\infty} f(x')\exp(-\mathrm{i}2\pi sx')\left[\int_{-\infty}^{\infty} g(y)\exp(\mathrm{i}2\pi sy)\,\mathrm{d}y\right]\mathrm{d}x'$$

$$= \int_{-\infty}^{\infty} f(x')\exp(-\mathrm{i}2\pi sx')\,\mathrm{d}x' \times \int_{-\infty}^{\infty} g(y)\exp(\mathrm{i}2\pi sy)\,\mathrm{d}y$$

$$= F(s)G(-s).$$
