Problem 45) a) $F(s) = \int_{-\infty}^{\infty} f(x)\exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x = \int_1^3 \exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x$

$$= \exp(-\mathrm{i}2\pi sx)/(-\mathrm{i}2\pi s)\big|_{x=1}^{3}$$

$$= [\exp(-\mathrm{i}6\pi s) - \exp(-\mathrm{i}2\pi s)]/(-\mathrm{i}2\pi s)$$

$$= \exp(-\mathrm{i}4\pi s)\,[\exp(-\mathrm{i}2\pi s) - \exp(+\mathrm{i}2\pi s)]/(-\mathrm{i}2\pi s)$$

$$= \exp(-\mathrm{i}4\pi s)\sin(2\pi s)/(\pi s) = 2\exp(-\mathrm{i}4\pi s)\,\mathrm{sinc}(2s).$$

b) $f(x) = \mathrm{rect}[(x-2)/2]$. Here, the standard $\mathrm{rect}(\cdot)$ function is shifted to the right by 2 units. Also, the division of the argument of the function by 2 doubles the width of the function. Overall, this is a rectangular function that equals 1.0 when x falls within the range 2 ± 1 (i.e., $1 \le x \le 3$), and is zero outside that range.

The shift theorem of Fourier transform asserts that $\mathcal{F}\{g(x-x_0)\} = \exp(-\mathrm{i}2\pi x_0 s)\,G(s)$, where $G(s) = \mathcal{F}\{g(x)\}$. Here, $x_0 = 2$ introduces the multiplicative phase-factor $\exp(-\mathrm{i}4\pi s)$. The scaling theorem of Fourier transform asserts that $\mathcal{F}\{g(x/\alpha)\} = |\alpha| G(\alpha s)$. Here, $\alpha = 2$ changes the Fourier transform $\mathrm{sinc}(s)$ of $\mathrm{rect}(x)$ to $2\mathrm{sinc}(2s)$ for the scaled version of the function, namely, $\mathrm{rect}(x/2)$. Thus,

$$F(s) = \mathcal{F}\{\mathrm{rect}[(x-2)/2]\} = \exp(-\mathrm{i}4\pi s)\,\mathcal{F}\{\mathrm{rect}(x/2)\} = \exp(-\mathrm{i}4\pi s)\,[2\,\mathrm{sinc}(2s)].$$

The results obtained for $F(s)$ in parts (a) and (b) are clearly identical.