Problem 44) This is a direct consequence of the Cauchy-Goursat theorem. Since $f(z)$ and $\exp(-\mathrm{i}2\pi sz)$ are analytic functions of z within the rectangular strip, the integral of their product around the closed contour must vanish. Considering that, in the limit when $L \to \infty$, the vertical lines do not contribute to the loop integral, the integral on the real axis x must be equal to that on the parallel line $z = x + \mathrm{i}y_0$; that is,

$\mathrm{d}z = \mathrm{d}(x + \mathrm{i}y_0) = \mathrm{d}x$

$$\int_{-\infty}^{\infty} f(x)\exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x = \int_{-\infty}^{\infty} f(x + \mathrm{i}y_0)\exp[-\mathrm{i}2\pi s(x + \mathrm{i}y_0)]\,\mathrm{d}z$$

$$\rightarrow \quad F(s) = \exp(2\pi y_0 s)\int_{-\infty}^{\infty} f(x + \mathrm{i}y_0)\exp(-\mathrm{i}2\pi sx)\,\mathrm{d}x$$

$$\rightarrow \quad \mathcal{F}\{f(x + \mathrm{i}y_0)\} = \exp(-2\pi y_0 s)\,F(s).$$