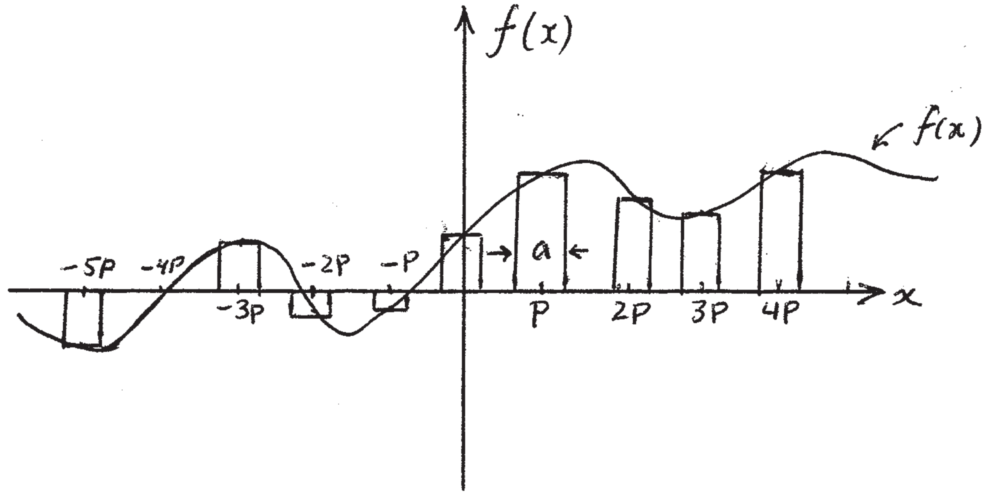
Problem 29)

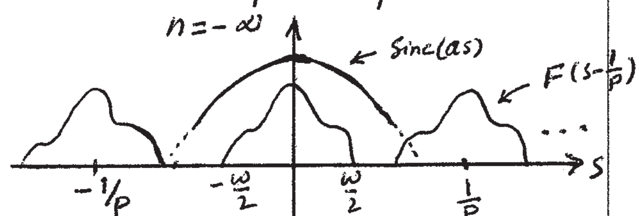


$$\hat{f}(x) = \sum_{n=-\infty}^{\infty} f(np)\,\mathrm{Rect}\left(\frac{x-np}{a}\right) = \mathrm{Rect}\left(\frac{x}{a}\right) * \left[\frac{1}{p}\,\mathrm{Comb}\left(\frac{x}{p}\right) f(x)\right]$$

$$\hat{F}(s) = \mathcal{F}\left\{\mathrm{Rect}\left(\frac{x}{a}\right)\right\} \cdot \mathcal{F}\left\{\frac{1}{p}\,\mathrm{Comb}\left(\frac{x}{p}\right) f(x)\right\} = a\,\mathrm{Sinc}(as)\left[\mathrm{Comb}(ps) * F(s)\right]$$

$$= a\,\mathrm{Sinc}(as) \sum_{n=-\infty}^{\infty} \delta(ps-n) * F(s) = a\,\mathrm{Sinc}(as) \sum_{n=-\infty}^{\infty} \frac{1}{p}\,\delta\left(s-\frac{n}{p}\right) * F(s) \Rightarrow$$

$$\hat{F}(s) = \frac{a}{p}\,\mathrm{Sinc}(as) \sum_{n=-\infty}^{\infty} F\left(s-\frac{n}{p}\right).$$



In the Fourier domain, the spectrum $F(s)$ of the original function $f(x)$ is repeated at intervals of $\frac{1}{p}$. Provided that $\frac{1}{p} \geq W$, i.e., the sampling rate $1/p$ is greater than the full bandwidth W of the function $f(x)$, the spectra remain isolated and there will be no aliasing. Aside from the constant scaling factor a/p, the spectrum will be multiplied by $\mathrm{Sinc}(as)$. The first zeros of this function are at $s = \pm 1/a$. Since $a \leq p$, we'll have $\frac{1}{a} \geq \frac{1}{p} \geq W$; therefore, the central spectrum corresponding to $n=0$ will not be affected by these zeros. A low-pass filter with transfer function $\frac{1}{\mathrm{Sinc}(as)}$, $-\frac{W}{2} < s < \frac{W}{2}$ will thus recover $f(x)$ from $\hat{f}(x)$.