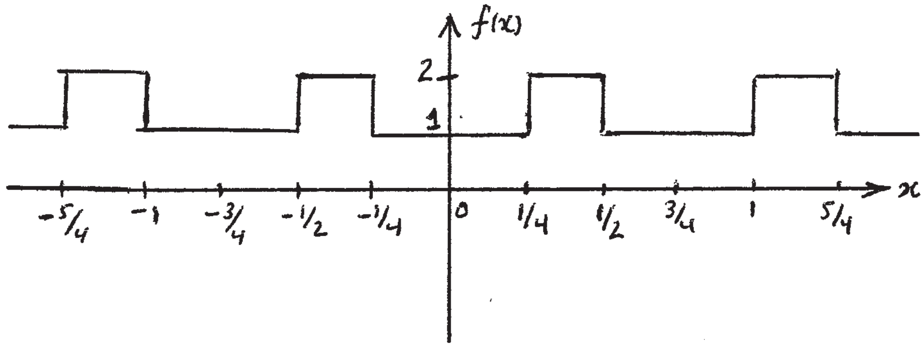
Problem 28)

a) To show that $f(x)$ is periodic with a period P, all we need to show is that $f(x+P)=f(x)$. We thus write:

$$f(x+P)=\sum_{n=-\infty}^{\infty}\underset{\sim}{f}(x+P-nP)=\sum_{n=-\infty}^{\infty}\underset{\sim}{f}\big(x-(n-1)P\big)=\sum_{m=-\infty}^{\infty}\underset{\sim}{f}(x-mP)=f(x) \checkmark$$

$m=n-1$

b)



$$C_0=\frac{1}{P}\int_0^P f(x)\,dx=\frac{4}{3}\left(\int_0^{1/4}f(x)dx+\int_{1/4}^{1/2}f(x)dx+\int_{1/2}^{3/4}f(x)dx\right)=\frac{4}{3}\left(\frac{1}{4}+\frac{2}{4}+\frac{1}{4}\right)=\frac{4}{3}$$

$$C_n=\frac{1}{P}\int_0^P f(x)e^{-i2\pi nx/P}dx=\frac{4}{3}\left\{\int_0^{1/4}e^{-i8\pi nx/3}dx+2\int_{1/4}^{1/2}e^{-i8\pi nx/3}dx+\int_{1/2}^{3/4}e^{-i8\pi nx/3}dx\right\}$$

$$=\frac{4}{3}\left(\frac{1}{-i8\pi n/3}\right)\left\{e^{-i2\pi n/3}\cancel{-1}+2e^{-i4\pi n/3}-2e^{-i2\pi n/3}+\cancel{e^{-i2\pi n}}-e^{-i4\pi n/3}\right\}$$

$$=\frac{i}{2\pi n}\left(e^{-i4\pi n/3}-e^{-i2\pi n/3}\right)=\frac{i}{2\pi n}\left(e^{+i2\pi n/3}-e^{-i2\pi n/3}\right)=-\frac{\sin(2\pi n/3)}{\pi n}$$

Digression: The function $f(x)$ may be written as $f(x)=2-\sum_{n=-\infty}^{\infty}\mathrm{Rect}[2(x-nP)]$;

See the plot of $f(x)$ above. The constant 2 will eventually be absorbed into C_0. The rest of the function is a periodic repetition of $-\mathrm{Rect}(2x)$, with period $P=3/4$. The F.T. of $-\mathrm{Rect}(2x)$ is $-\frac{1}{2}\mathrm{Sinc}(\frac{s}{2})$. Sampling this function at the rate of $\frac{1}{P}=\frac{4}{3}$ yields

$$C_n=\frac{1}{P}\underset{\sim}{F}\left(\frac{n}{P}\right)=\frac{4}{3}\left(-\frac{1}{2}\right)\mathrm{Sinc}\left(\frac{2n}{3}\right)=-\frac{2}{3}\,\frac{\sin(2\pi n/3)}{2\pi n/3}=-\frac{\sin(2\pi n/3)}{\pi n}$$, as before. Also, $C_0=2-\frac{4}{3}\left[\frac{1}{2}\mathrm{Sinc}(0)\right]=\frac{4}{3}$. ✓