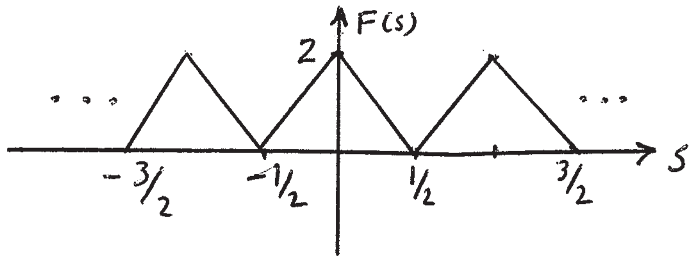
Problem 25) a) $f(x) = \text{sinc}^2(x/2)\,\text{Comb}(x)$

$\mathcal{F}\{\text{sinc}^2(x)\} = \text{Tri}(s) \Rightarrow \mathcal{F}\{\text{sinc}^2(x/2)\} = 2\,\text{Tri}(2s)$.

Scaling property

$\mathcal{F}\{\text{Comb}(x)\} = \text{Comb}(s)$

Therefore, $\mathcal{F}\{f(x)\} = \mathcal{F}\{\text{sinc}^2(x/2)\} * \mathcal{F}\{\text{Comb}(x)\} = 2\,\text{Tri}(2s) * \text{Comb}(s)$



The area under $f(x)$ is thus equal to $F(0) = 2$. We have:

$$\int_{-\infty}^{\infty} \text{sinc}^2\left(\frac{x}{2}\right)\text{Comb}(x)\,dx = \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n}{2}\right) = \text{sinc}^2(0) + 2\sum_{n=1}^{\infty} \text{sinc}^2\left(\frac{n}{2}\right)$$

$$= 1 + 2\sum_{n=1}^{\infty} \frac{\sin^2(n\pi/2)}{(n\pi/2)^2} = 1 + 2\sum_{n=1,3,5,7,\dots}^{\infty} \frac{1}{(n\pi/2)^2} = 1 + \frac{8}{\pi^2}\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} = 2$$

$$\Rightarrow \sum_{n=1,3,5,7,\dots}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{8}$$

b) $$\sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=1}^{\infty} \frac{1}{(2n)^2} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \left(1-\frac{1}{4}\right)\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{4}{3}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{4}{3}\cdot\frac{\pi^2}{8} = \frac{\pi^2}{6}.$$ ✓