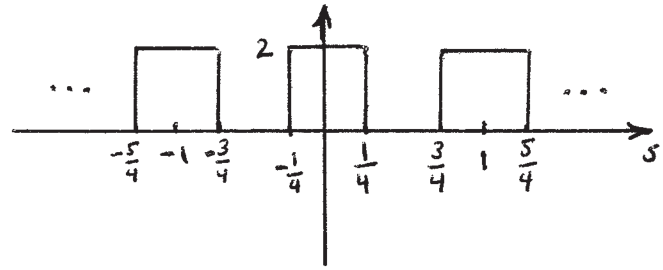
Problem 24) $\mathcal{F}\{\text{Sinc}(\frac{x}{2})\,\text{Comb}(x)\} = \mathcal{F}\{\text{Sinc}(\frac{x}{2})\} * \mathcal{F}\{\text{Comb}(x)\}$

$= 2\,\text{Rect}(2s) * \text{Comb}(s) \longrightarrow$



Convolution with the Comb(·) function moves the rectangular pulse $2\,\text{Rect}(2s)$ to the location of each delta-function.

Now, the area under the function $\text{Sinc}(x/2)\,\text{Comb}(x)$ is equal to the value of the Fourier Transform at $s=0$. This value is equal to 2.

Therefore,

$$\int_{-\infty}^{\infty} \text{Sinc}(\tfrac{x}{2})\,\text{Comb}(x)\,dx = 2 \Rightarrow \int_{-\infty}^{\infty} \text{Sinc}(\tfrac{x}{2}) \sum_{n=-\infty}^{\infty} \delta(x-n)\,dx = 2 \Rightarrow$$

$$\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \text{Sinc}(\tfrac{x}{2})\,\delta(x-n)\,dx = \sum_{n=-\infty}^{\infty} \text{Sinc}(\tfrac{n}{2}) = \sum_{n=-\infty}^{\infty} \frac{\sin(n\pi/2)}{n\pi/2} = 2$$

Sifting Property of $\delta(x-n)$

At $n=0$, the value of the Sinc function is equal to 1. Also $\text{Sinc}(x)$ is an even function, therefore, its values at $+n$ and $-n$ are equal. We thus have:

$$1 + 2\sum_{n=1}^{\infty} \sin(n\pi/2)/(n\pi/2) = 2 \Rightarrow \frac{\sin(\pi/2)}{\pi/2} + \cancelto{0}{\frac{\sin(\pi)}{\pi}} + \frac{\sin(3\pi/2)}{3\pi/2} + \cancelto{0}{\frac{\sin(2\pi)}{2\pi}} + \cdots = \frac{1}{2}$$

$$\Rightarrow \frac{2}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) = \frac{1}{2} \Rightarrow 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

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