Problem 23) a) $f(x)=\sum_{n=-\infty}^{\infty}\mathrm{Rect}\left(\frac{x-np}{a}\right)=\frac{1}{p}\mathrm{Comb}\left(\frac{x}{p}\right)*\mathrm{Rect}\left(\frac{x}{a}\right)$

$$\Rightarrow F(s)=\mathcal{F}\left\{\frac{1}{p}\mathrm{Comb}\left(\frac{x}{p}\right)\right\}\cdot\mathcal{F}\left\{\mathrm{Rect}\left(\frac{x}{a}\right)\right\}=\mathrm{Comb}(ps)\cdot a\,\mathrm{Sinc}(as)$$

$$=a\,\mathrm{Sinc}(as)\sum_{n=-\infty}^{\infty}\delta(ps-n)=a\,\mathrm{Sinc}(as)\sum_{n=-\infty}^{\infty}\frac{1}{p}\delta\left(s-\frac{n}{p}\right)\Rightarrow$$

$$F(s)=\frac{a}{p}\sum_{n=-\infty}^{\infty}\mathrm{Sinc}\left(\frac{an}{p}\right)\delta\left(s-\frac{n}{p}\right).$$

Therefore, $f(x)=\int_{-\infty}^{\infty}F(s)e^{+i2\pi sx}ds=\frac{a}{p}\sum_{n=-\infty}^{\infty}\mathrm{Sinc}\left(\frac{an}{p}\right)\int_{-\infty}^{\infty}\delta\left(s-\frac{n}{p}\right)e^{i2\pi sx}ds$

$$\Rightarrow f(x)=\frac{a}{p}\sum_{n=-\infty}^{\infty}\mathrm{Sinc}\left(\frac{an}{p}\right)e^{+i2\pi nx/p}=\frac{a}{p}+\frac{2a}{p}\sum_{n=1}^{\infty}\mathrm{Sinc}\left(\frac{an}{p}\right)\mathrm{Cos}\left(\frac{2\pi nx}{p}\right).$$

b) $g(x)=\sum_{n=-\infty}^{\infty}\mathrm{Tri}\left(\frac{x-np}{a}\right)=\frac{1}{p}\mathrm{Comb}\left(\frac{x}{p}\right)*\mathrm{Tri}\left(\frac{x}{a}\right)\Rightarrow$

$$G(s)=\mathrm{Comb}(ps)\cdot a\,\mathrm{Sinc}^2(as)=a\,\mathrm{Sinc}^2(as)\sum_{n=-\infty}^{\infty}\delta(ps-n)\Rightarrow$$

$$G(s)=\frac{a}{p}\sum_{n=-\infty}^{\infty}\mathrm{Sinc}^2\left(\frac{an}{p}\right)\delta\left(s-\frac{n}{p}\right).$$

Therefore, $g(x)=\int_{-\infty}^{\infty}G(s)e^{+i2\pi sx}ds=\frac{a}{p}\sum_{n=-\infty}^{\infty}\mathrm{Sinc}^2\left(\frac{an}{p}\right)\int_{-\infty}^{\infty}\delta\left(s-\frac{n}{p}\right)e^{i2\pi sx}ds$

$$\Rightarrow g(x)=\frac{a}{p}\sum_{n=-\infty}^{\infty}\mathrm{Sinc}^2\left(\frac{an}{p}\right)e^{i2\pi nx/p}=\frac{a}{p}+\frac{2a}{p}\sum_{n=-\infty}^{\infty}\mathrm{Sinc}^2\left(\frac{an}{p}\right)\mathrm{Cos}\left(\frac{2\pi nx}{p}\right).$$