Problem 21) $f(x) = x\,Rect(x/2)$

$$\mathcal{F}\{Rect(x)\} = Sinc(s) \xrightarrow[\text{theorem}]{\text{Scaling}} \mathcal{F}\{Rect(x/2)\} = 2\,Sinc(2s) = \frac{Sin(2\pi s)}{\pi s}$$

$$G(s) = \int_{-\infty}^{\infty} g(x) e^{-i2\pi s x}\,dx \Rightarrow \frac{dG(s)}{ds} = \int_{-\infty}^{\infty} -i2\pi x\, g(x) e^{-i2\pi s x}\,dx \Rightarrow$$

$$\mathcal{F}\{x\,g(x)\} = \frac{i}{2\pi}\frac{dG(s)}{ds}$$ ← differentiation theorem

Therefore, $\mathcal{F}\{x\,Rect(x/2)\} = \frac{i}{2\pi}\frac{d}{ds}\left[\frac{Sin(2\pi s)}{\pi s}\right] = \frac{i}{2\pi^2}\,\frac{2\pi s\,Cos(2\pi s) - Sin(2\pi s)}{s^2}$

$$\Rightarrow \mathcal{F}\{x\,Rect(x/2)\} = \frac{i\,Cos(2\pi s)}{\pi s} - \frac{i\,Sin(2\pi s)}{2\pi^2 s^2}.$$

$$F(s) = \mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-i2\pi s x}\,dx = \int_{-1}^{+1} x\, e^{-i2\pi s x}\,dx$$

integration by Parts

$$= \frac{x}{-i2\pi s} e^{-i2\pi s x}\Big|_{x=-1}^{+1} + \frac{1}{i2\pi s}\int_{-1}^{+1} e^{-i2\pi s x}\,dx$$

$$= \frac{i}{2\pi s}\left(e^{-i2\pi s} + e^{+i2\pi s}\right) - \frac{1}{(i2\pi s)^2}\, e^{-i2\pi s x}\Big|_{x=-1}^{+1}$$

$$= \frac{i\,Cos(2\pi s)}{\pi s} + \frac{1}{4\pi^2 s^2}\left(e^{-i2\pi s} - e^{+i2\pi s}\right)$$

$$= \frac{i\,Cos(2\pi s)}{\pi s} - \frac{i\,Sin(2\pi s)}{2\pi^2 s^2}.$$