

Problem 20)

a) $f(x) = \text{Tri}(x)$.

b) $F(s) = \int_{-\infty}^{\infty} f(x) \exp(-i2\pi sx) dx = \frac{1}{2} \int_0^2 \int_{-\infty}^{\infty} \text{Rect}(x/\beta) \exp(-i2\pi sx) dx d\beta$

Using scaling property
and the fact that
 $\mathcal{F}\{\text{Rect}(x)\} = \text{sinc}(s)$.

$$\rightarrow = \frac{1}{2} \int_0^2 \beta \text{sinc}(\beta s) d\beta = \frac{1}{2} \int_0^2 \beta \frac{\sin(\pi\beta s)}{\pi\beta s} d\beta = \frac{1}{2\pi s} \int_0^2 \sin(\pi\beta s) d\beta$$

$$= -\frac{\cos(\pi\beta s)}{2(\pi s)^2} \Big|_{\beta=0}^2 = \frac{1 - \cos(2\pi s)}{2(\pi s)^2} = \frac{2 \sin^2(\pi s)}{2(\pi s)^2} = \text{sinc}^2(s).$$

It is thus confirmed that the Fourier transform of $f(x)$ is the same as that of $\text{Tri}(x)$.