Problem 19) a) $\mathscr{F}\{\cos(2\pi f_o x)\} = \tfrac{1}{2}\mathscr{F}\{\exp(\mathrm{i}2\pi f_o x) + \exp(-\mathrm{i}2\pi f_o x)\} = \tfrac{1}{2}[\delta(s - f_o) + \delta(s + f_o)].$

b) $\mathscr{F}\{\sin(2\pi f_o x)\} = \tfrac{1}{2\mathrm{i}}\mathscr{F}\{\exp(\mathrm{i}2\pi f_o x) - \exp(-\mathrm{i}2\pi f_o x)\} = \tfrac{1}{2\mathrm{i}}[\delta(s - f_o) - \delta(s + f_o)].$

c) $\mathscr{F}\{\cos^2(\pi f_o x)\} = \tfrac{1}{2}\mathscr{F}\{1 + \cos(2\pi f_o x)\} = \tfrac{1}{2}\delta(s) + \tfrac{1}{4}[\delta(s - f_o) + \delta(s + f_o)].$

d) Differentiation theorem:

$$f(x) = \int_{-\infty}^{\infty} F(s)\exp(\mathrm{i}2\pi sx)\mathrm{d}s \quad \rightarrow \quad \frac{\mathrm{d}f(x)}{\mathrm{d}x} = \int_{-\infty}^{\infty} \mathrm{i}2\pi sF(s)\exp(\mathrm{i}2\pi sx)\mathrm{d}s \quad \rightarrow \mathscr{F}\left\{\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right\} = \mathrm{i}2\pi sF(s).$$

Therefore,

$$\mathscr{F}\left\{\frac{\mathrm{d}}{\mathrm{d}x}\cos^2(\pi f_o x)\right\} = \mathrm{i}2\pi s\left\{\tfrac{1}{2}\delta(s) + \tfrac{1}{4}[\delta(s - f_o) + \delta(s + f_o)]\right\} = 0 + \frac{\mathrm{i}2\pi}{4}[f_o\delta(s - f_o) - f_o\delta(s + f_o)].$$

Carrying out the differentiation, we find

$$\mathscr{F}\{-2\pi f_o \sin(\pi f_o x)\cos(\pi f_o x)\} = -\pi f_o \mathscr{F}\{\sin(2\pi f_o x)\} = \frac{\mathrm{i}\pi f_o}{2}[\delta(s - f_o) - \delta(s + f_o)].$$

Consequently,

$$\mathscr{F}\{\sin(2\pi f_o x)\} = \tfrac{1}{2\mathrm{i}}[\delta(s - f_o) - \delta(s + f_o)].$$

This is the same result as obtained in part (b).