

Problem 17) Method 1:

$$\begin{aligned}F(s) &= \int_{-\infty}^{\infty} f(x) \exp(-i2\pi sx) dx = \int_{-3/4}^{3/4} \cos(2\pi x) \exp(-i2\pi sx) dx \\&= \frac{1}{2} \int_{-3/4}^{3/4} [\exp(i2\pi x) + \exp(-i2\pi x)] \exp(-i2\pi sx) dx \\&= \frac{1}{2} \int_{-3/4}^{3/4} \{\exp[i2\pi(1-s)x] + \exp[-i2\pi(1+s)x]\} dx \\&= \frac{\exp[i(3\pi/2)(1-s)] - \exp[-i(3\pi/2)(1-s)]}{i4\pi(1-s)} + \frac{\exp[-i(3\pi/2)(1+s)] - \exp[i(3\pi/2)(1+s)]}{-i4\pi(1+s)} \\&= \frac{2i \sin[(3\pi/2)(1-s)]}{i4\pi(1-s)} + \frac{-2i \sin[(3\pi/2)(1+s)]}{-i4\pi(1+s)} = \frac{3}{4} \{\text{sinc}[3(s-1)/2] + \text{sinc}[3(s+1)/2]\}.\end{aligned}$$

Method 2: $f(x) = \text{Rect}(2x/3) \cos(2\pi x)$. Therefore, using the scaling and convolution theorems along with the fact that $\mathcal{F}\{\exp(\pm i2\pi s_0 x)\} = \delta(s \mp s_0)$, we will have

$$\begin{aligned}F(s) &= \frac{3}{2} \text{sinc}(3s/2) * \frac{1}{2} [\delta(s-1) + \delta(s+1)] \\&= \frac{3}{4} \{\text{sinc}[3(s-1)/2] + \text{sinc}[3(s+1)/2]\}.\end{aligned}$$
