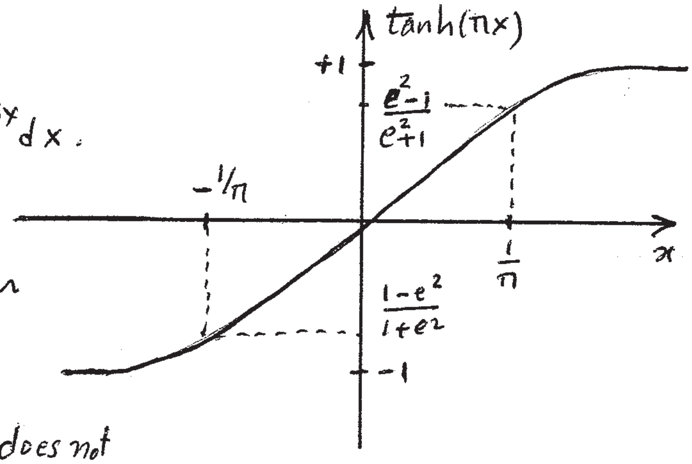
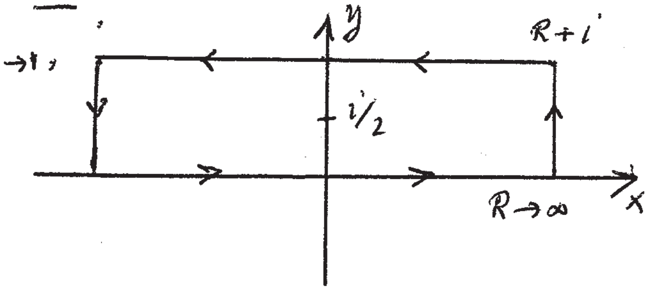
Problem 15)



$$\mathcal{F}\{\tanh(\pi x)\} = \int_{-\infty}^{\infty} \frac{\sinh(\pi x)}{\cosh(\pi x)} e^{-i2\pi s x} dx.$$

We use the rectangular contour shown below. On the vertical sides of the rectangle, the integral does not go to zero as $R \to \infty$. Here $\tanh(\pi z) \to 1$, and the integrals on the vertical legs approach $\pm \int_0^1 e^{\mp i2\pi R s + 2\pi s y} i\,dy$



$= \pm \frac{i e^{\mp i2\pi R s}}{2\pi s}(e^{2\pi s} - 1)$. The total contribution of the vertical legs is thus given by $(e^{2\pi s} - 1)\frac{\sin(2\pi R s)}{\pi s}$. As $R \to \infty$, this becomes a rapidly oscillating function of s, which has no significant physical effects, and is typically ignored.

Residue at $z_0 = i/2$: $\frac{\sinh(i\pi/2)}{\pi \sinh(i\pi/2)} e^{-i2\pi s(i/2)} = \frac{e^{\pi s}}{\pi}$.

Cauchy's theorem then yields:

$$\int_{-\infty}^{\infty} \frac{\sinh(\pi x)}{\cosh(\pi x)} e^{-i2\pi s x} dx - \int_{-\infty}^{\infty} \frac{\sinh(\pi x + i\pi)}{\cosh(\pi x + i\pi)} e^{-i2\pi s(x+i)} dx = 2\pi i \frac{e^{\pi s}}{\pi}$$

$$\Rightarrow (1 - e^{2\pi s}) \int_{-\infty}^{\infty} \tanh(\pi x) e^{-i2\pi s x} dx = 2i e^{\pi s} \Rightarrow$$

$$\mathcal{F}\{\tanh(\pi x)\} = \frac{2i e^{\pi s}}{1 - e^{2\pi s}} = \frac{2i}{e^{-\pi s} - e^{\pi s}} = -\frac{i}{\sinh(\pi s)}.$$