Problem 13) We use the scaling theorem, $\mathcal{F}\{f(\alpha x)\} = \frac{1}{|\alpha|} F(\frac{s}{\alpha})$ to obtain the F.T. of $\operatorname{Sinc}(ax)$ as $\frac{1}{a}\operatorname{Rect}(\frac{s}{a})$, and also the F.T. of $e^{-b|x|}$ as $\frac{1}{b}\frac{2}{1+(2\pi s/b)^2} = \frac{2b}{b^2+(2\pi s)^2}$. (Note that $a>0$ and $b>0$).

Next, we use the convolution theorem to write:

$$\mathcal{F}\{\operatorname{sinc}(ax)e^{-b|x|}\} = \frac{1}{a}\operatorname{Rect}(\frac{s}{a}) * \frac{2b}{b^2+(2\pi s)^2} = \frac{2b}{a}\int_{-\infty}^{\infty}\frac{1}{b^2+(2\pi s')^2}\operatorname{Rect}(\frac{s-s'}{a})ds'$$

$$= \frac{2b}{a}\int_{s-a/2}^{s+a/2}\frac{ds'}{b^2+(2\pi s')^2}$$

At this point we change the variable from s' to θ such that $2\pi s' = b\tan\theta$. We'll have

$$\mathcal{F}\{\operatorname{Sinc}(ax)e^{-b|x|}\} = \frac{2b}{a}\int_{\tan^{-1}[\frac{2\pi}{b}(s-\frac{a}{2})]}^{\tan^{-1}[\frac{2\pi}{b}(s+\frac{a}{2})]}\frac{1}{b^2+b^2\tan^2\theta}\,\frac{b(1+\tan^2\theta)}{2\pi}\,d\theta$$

$$= \frac{1}{\pi a}\left\{\tan^{-1}\left[\frac{2\pi}{b}(s+\frac{a}{2})\right] - \tan^{-1}\left[\frac{2\pi}{b}(s-\frac{a}{2})\right]\right\}$$

Taking the tangent of both sides of the above equation, and using the trig identity $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$, yields:

$$\tan\{\pi a\mathcal{F}\{\operatorname{sinc}(ax)e^{-b|x|}\}\} = \frac{\frac{2\pi}{b}(s+\frac{a}{2})-\frac{2\pi}{b}(s-\frac{a}{2})}{1+(\frac{2\pi}{b})^2(s+\frac{a}{2})(s-\frac{a}{2})} = \frac{2\pi a/b}{1+(\frac{2\pi}{b})^2(s^2-\frac{a^2}{4})}$$

$$= \frac{\pi ab}{2\pi^2s^2+\frac{1}{2}b^2-\frac{1}{2}\pi^2a^2} \Rightarrow \pi a\mathcal{F}\{\operatorname{sinc}(ax)e^{-b|x|}\} = \tan^{-1}\left\{\frac{\pi ab}{2\pi^2s^2+\frac{1}{2}(b^2-\pi^2a^2)}\right\}.$$