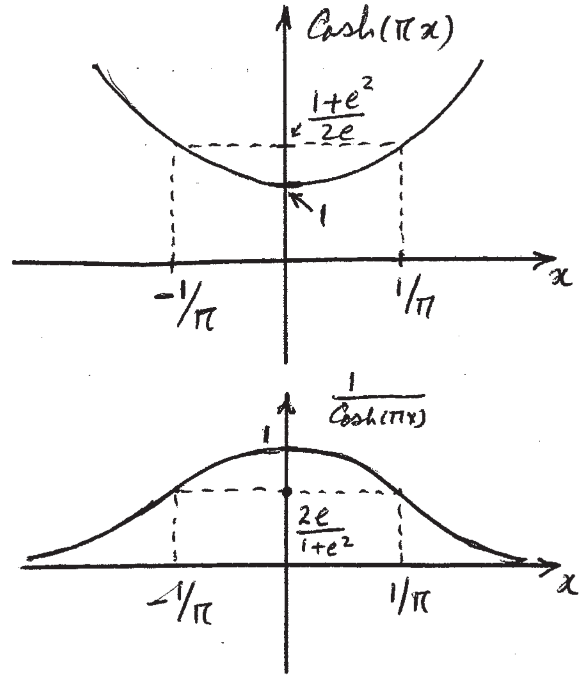
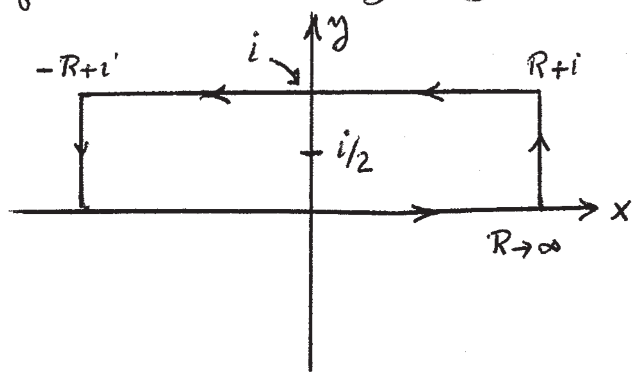
Problem 9)



$$\mathcal{F}\left\{\frac{1}{\cosh(\pi x)}\right\} = \int_{-\infty}^{\infty} \frac{e^{-i2\pi s x}}{\cosh(\pi x)}\,dx$$

The Contour Chosen for this integral in the Complex-plane is shown below. As $R \to \infty$ the Contributions of the Vertical lines go to Zero.



On the horizontal line $z = x+i$, we have $\cosh(\pi z) = \dfrac{e^{\pi(x+i)} + e^{-\pi(x+i)}}{2}$

$$= \frac{e^{i\pi} e^{\pi x} + e^{-i\pi} e^{-\pi x}}{2} = -\frac{e^{\pi x} + e^{-\pi x}}{2} = -\cosh(\pi x).$$

Also, on this horizontal line, $e^{-i2\pi s z} = e^{-i2\pi s(x+i)} = e^{2\pi s} e^{-i2\pi s x}$.

The poles of the integral are the zeros of $\cosh(\pi z)$, namely, $z_n = (n+\frac{1}{2})i$. The only pole inside the above Contour is $z_0 = \frac{i}{2}$. To find the residue of the integral at this pole we write:

$$\cosh(\pi z) = \cosh(\pi i/2) + \pi \sinh(\pi i/2)(z - z_0) + \cdots = \pi i (z - z_0) + \cdots$$

The integral over the Closed Contour must be $2\pi i$ times the residue at $z = z_0$. We'll have

$$\int_{-\infty}^{\infty} \frac{e^{-i2\pi s x}}{\cosh(\pi x)}\,dx - \int_{-\infty}^{\infty} \frac{e^{2\pi s}\, e^{-i2\pi s x}}{-\cosh(\pi x)}\,dx = 2\pi i\, \frac{e^{-i2\pi s(\frac{i}{2})}}{i\pi} = 2e^{\pi s} \Rightarrow$$

$$\mathcal{F}\left\{\frac{1}{\cosh(\pi x)}\right\} = \frac{2e^{\pi s}}{1+e^{2\pi s}} = \frac{2}{e^{-\pi s}+e^{\pi s}} = \frac{1}{\cosh(\pi s)}.$$