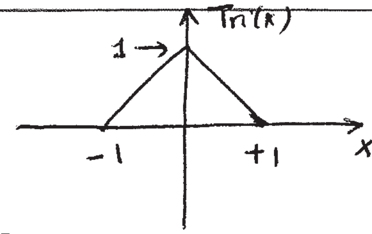
Problem 2)

$$\mathrm{Tri}(x) = \begin{cases} 1-|x|; & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$



$$\mathrm{Rect}(x) * \mathrm{Rect}(x) = \int_{-\infty}^{\infty} \mathrm{Rect}(x')\,\mathrm{Rect}(x-x')\,dx' = \int_{-1/2}^{1/2} \mathrm{Rect}(x-x')\,dx' =$$

Change of Variable $y = x - x'$

$$-\int_{x+1/2}^{x-1/2} \mathrm{Rect}(y)\,dy = \int_{x-1/2}^{x+1/2} \mathrm{Rect}(y)\,dy$$

Now, if $x<-1$ or $x>+1$, the range of the above integral will be outside the range where $\mathrm{Rect}(y)$ is nonzero. Therefore, the integral will vanish when $|x|>1$. However, if x is in the interval $(-1, 0)$, the lower-limit of the integral will be to the left of the rectangle, while its upper limit will be somewhere within the rectangle. We'll then have:

$$\mathrm{Tri}(x) = \int_{-1/2}^{x+1/2} \mathrm{Rect}(y)\,dy = \int_{-1/2}^{x+1/2} dy = y\Big|_{-1/2}^{x+1/2} = x+\tfrac{1}{2}-(-\tfrac{1}{2}) = 1+x; \quad -1 \leq x \leq 0.$$

When x is in the range $(0, +1)$, the lower limit of the integral will be somewhere inside the rectangle, while its upper limit will be to the right of the rectangle. We'll have

$$\int_{x-1/2}^{1/2} \mathrm{Rect}(y)\,dy = \int_{x-1/2}^{1/2} dy = y\Big|_{x-1/2}^{1/2} = \tfrac{1}{2} - (x-\tfrac{1}{2}) = 1-x; \qquad 0 \leq x \leq 1.$$

Combining the above results, it is clear that $\mathrm{Tri}(x) = 1-|x|$ when $|x| \leq 1$, and zero otherwise.

Using the convolution theorem, we may write:

$$\mathcal{F}\{\mathrm{Tri}(x)\} = \mathcal{F}\{\mathrm{Rect}(x) * \mathrm{Rect}(x)\} = \mathcal{F}\{\mathrm{Rect}(x)\}\cdot\mathcal{F}\{\mathrm{Rect}(x)\} = \mathrm{Sinc}^2(s).$$