

Problem 1)

$$\begin{aligned}
 H(s) = \mathcal{F}\{f(x) * g(x)\} &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(x') g(x-x') dx' \right] e^{-i2\pi s x} dx = \\
 &= \int_{-\infty}^{\infty} f(x') \int_{-\infty}^{\infty} g(x-x') e^{-i2\pi s x} dx dx' = \int_{-\infty}^{\infty} f(x') \int_{-\infty}^{\infty} g(y) e^{-i2\pi s(x'+y)} dy dx' = \\
 &= \int_{-\infty}^{\infty} f(x') e^{-i2\pi s x'} \int_{-\infty}^{\infty} g(y) e^{-i2\pi s y} dy dx' = F(s) G(s). \checkmark
 \end{aligned}$$

↑ Change of variable: $y = x - x'$

$$\begin{aligned}
 f(x) * g(x) &= \int_{-\infty}^{\infty} f(x') g(x-x') dx' = \int_{-\infty}^{\infty} f(x-y) g(y) dy = g(x) * f(x). \checkmark
 \end{aligned}$$

↑ Change of variable
 $y = x - x'$