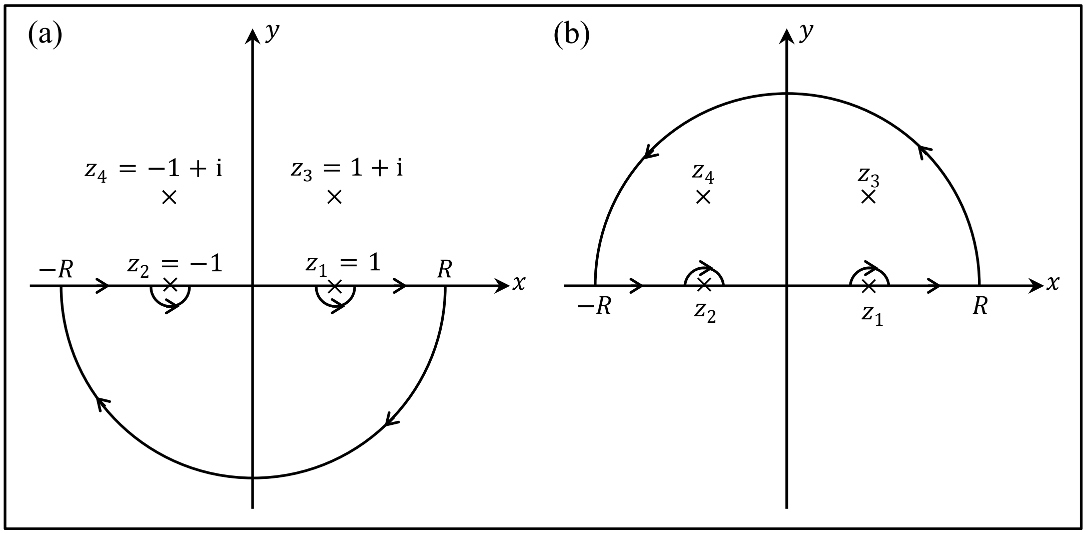
Problem 42) The poles of the integrand in the complex z-plane are readily found to be

$$(z^2-1)(z^2-2iz-2)=0 \quad \rightarrow \quad z_{1,2}=\pm 1 \quad \text{and} \quad z_{3,4}=i\pm\sqrt{i^2+2}=\pm 1+i.$$

Both contours of integration depicted in Figs.(a) and (b) are acceptable, since, in each case, the integral over the large semi-circle goes to zero when $R \to \infty$.



In Fig.(a) the contour is closed in the lower-half of the z-plane. The closed contour does *not* contain any poles and, therefore, the integral around the closed loop is zero. This means that the desired integral equals the sum of the half-residues at $z=z_1$ and $z=z_2$. The residues at $z_{1,2}$ are

$$\left.\frac{1}{(z+1)(z^2-2iz-2)}\right|_{z=z_1}=\frac{1}{2(1-2i-2)}=-\frac{1}{2(1+2i)}=-\frac{1-2i}{2(1+4)}=-0.1+0.2i,$$

$$\left.\frac{1}{(z-1)(z^2-2iz-2)}\right|_{z=z_2}=\frac{1}{-2(1+2i-2)}=\frac{1}{2(1-2i)}=\frac{1+2i}{2(1+4)}=0.1+0.2i.$$

The sum of the residues at z_1 and z_2 is thus seen to be $0.4i$. This must be multiplied by $-i\pi$, where the minus sign accounts for the counterclockwise direction of rotation around the small semi-circles in Fig.(a). The desired integral is thus equal to 0.4π.

In Fig.(b) the closed contour contains the poles at z_3 and z_4. The residues at these poles are

$$\left.\frac{1}{(z^2-1)(z+1-i)}\right|_{z=z_3}=\frac{1}{[(1+i)^2-1][(1+i)+1-i]}=\frac{1}{2(-1+2i)}=-\frac{1+2i}{2(1+4)}=-0.1-0.2i,$$

$$\left.\frac{1}{(z^2-1)(z-1-i)}\right|_{z=z_4}=\frac{1}{[(-1+i)^2-1][(-1+i)-1-i]}=\frac{1}{2(1+2i)}=\frac{1-2i}{2(1+4)}=0.1-0.2i.$$

The sum of the residues at z_3 and z_4 is, therefore, $-0.4i$, which, upon multiplying with $2\pi i$, yields 0.8π. To this we must now add the sum of the half-residues at z_1 and z_2. This will be the same as the result obtained in the previous case except for a change of sign — because the direction of travel around the small semi-circles in Fig.(b) is clockwise. Therefore, the desired integral is given by $0.8\pi - 0.4\pi = 0.4\pi$, as before.